En10-A A Finite Enter Instrument on a Qubit–Antiqubit Interface: Classical Records, Source-Weight Estimation, and Non-Identification

Ngo Cheung

Abstract

We construct a complete finite instrument for a two-zone qubit–antiqubit toy model. The antiqubit designation means a declared inverse response to one specified unitary generator; it does not introduce antimatter, an antiunitary operation, or backward physical time. Both data qubits are read destructively in the computational basis. The resulting channel returns a classical word, consumes the corresponding quantum names, and agrees exactly with its projective recorded-channel specification on all operators, including arbitrary untouched references. For an independently supplied one-parameter source family, a separate classical inference map estimates the normalized source weight. Its finite-shot confidence radius, sensitivity denominator, Fisher-information ledger, and implementation-error allowance are explicit. A concrete instance has entirely rational outcome probabilities and estimator values. We also prove a record-only quantum-recovery obstruction and non-identifiability of parameters omitted from the frozen model. The philosophical contribution is an application of Enter's interface discipline: a record is not a quantum return; an inference map is not the record; an inaccessible complement is not destroyed information; a channel is not a thermal machine; and a finite analog is not an astrophysical identification. No Galactic quantum state, 511keV morphology, absolute Galactic rate, continuum transport law, or spacetime geometry is reconstructed.

Bibliographic record

Title	En10-A A Finite Enter Instrument on a Qubit–Antiqubit Interface: Classical Records, Source-Weight Estimation, and Non-Identification
Author	Cheung, Ngo
Publication date	2026
Institution	Enter
Report number	10-En10-A
Publisher	Cheung Ngo Medical Limited
First page	1
Last page	18
Language	en

Identifier and provenance. The identifier **En10-A** is used here as requested for this finite-model specialization. The separately supplied En10-A.tex, version 1, concerns relational clock readout [CheungRelational]. References to that source are written as *source En10-A@1*; references to the present main theorem use Theorem . This document neither replaces that source nor asserts that it is a registered revision of it.

Mathematical status. All mathematical claims needed for the instrument and estimation contract are proved below from explicitly stated finite-dimensional and statistical assumptions. The companion manuscripts are comparisons, not substitutes for these proofs. Born measurement semantics are assumed here; their characterization within Cheung's stated operational class is not independently re-proved.

Motivation and the philosophical background of Enter

From a physical analogy to an operational question

The Galactic positron problem motivates questions about sources, transport, and annihilation observables. Actual analyses of the 511keV signal involve instrumental backgrounds, spatial models, spectra, and component-dependent inference [Siegert2016]. None of those structures is supplied by the four-dimensional data Hilbert space used here.

The present question is consequently narrower: Given a declared finite preparation family and a declared finite channel, what does destructive readout return, and what may a separately declared statistical procedure infer from its records? The words ``bulge'' and ``disk'' label two basis values. They do not identify those values with regions of the Milky Way.

The term *antiqubit* is motivated by the platform-specific unitary-inversion construction of Song and collaborators [Song2025]. Their work distinguishes an inverse unitary response from the ordinary response of a qubit. Here that distinction is installed as a mathematical input. We do not construct their transmon controls or import their experimental metrological performance.

Enter as an interface discipline

Cheung's Enter calculus treats runtime quantum names as access rights and destructive computational-basis measurement as a consuming operation. The published article separates concrete Born semantics from an operational characterization within a specified strategy class [CheungEnter,CheungI].

The introductory essay *The Philosophy of Enter* emphasizes the distinctions between destructive readout, quantum recovery, implementation ledgers, and physical identification [CheungPhilosophy]. We use that essay as philosophical orientation, not as a theorem premise.

For the present construction, this orientation has five precise consequences.

• **A quantum name is an operational entitlement, not an ontology of the apparatus.** Consumption means that a continuation cannot use the measured name as an available quantum input. It does not mean that every microscopic carrier, environmental correlation, or physical degree of freedom has ceased to exist.

• **A classical record is not a quantum return.** The output word belongs to a commutative record algebra. It may be copied or used for classical control, but that does not restore arbitrary coherence or reference entanglement of the measured register.

• **Inference requires an additional contract.** A recorded word does not estimate a parameter by itself. The preparation family, likelihood, calibration constants, sample count, and inference map must be declared. Calling that map a statistical decoder does not make it a quantum-return decoder.

• **The observation boundary is an access boundary.** It specifies which record, spectator, and residual observables may be inspected and which continuations are permitted. It is not a light cone, a distance, or a Galactic radius.

• **An analog requires a separate identification theorem.** A finite channel may be useful without being the interstellar medium or a continuum transport law. Physical interpretation cannot be strengthened merely by changing the labels on its basis states.

These are methodological distinctions with mathematical consequences below. They are not a new measurement postulate, a claim about consciousness, or a resolution of every interpretational question in quantum mechanics.

Corrections to the proposed theorem

Three qualifications are necessary before a valid theorem can be stated.

Residuals can sometimes continue. Destructive readout of one factor does not imply that every unmeasured factor becomes incapable of further evolution. Source En10-A@1 explicitly retains a conditional system, and under additional covariance assumptions that system has a relational unitary evolution [CheungRelational]. The correct statement here is that *neither measured data name is returned*. Any surviving spectator or inaccessible environment is a different interface.

Two qubits do not encode every proposed observable. Two position qubits cannot simultaneously provide independent position, two-particle spin, energy-bin, and annihilation-flag registers. We therefore freeze a

position-only interface. The relaxation channel below is a declared surrogate, not a physical annihilation event. The primary records are occupation words, not annihilation or escape counts.

Identifiability is a hypothesis, not a consequence of readout. An arbitrary finite measurement need not distinguish a proposed source parameter. We select one family for which the accessible probability is affine with a strictly positive slope. The slope appears explicitly in every estimation bound.

The frozen finite interface

Registers, basis, and externally supplied source family

Let

$$\mathscr{H}_N = \mathbb{C}^2, \qquad \mathscr{H}_A = \mathbb{C}^2, \qquad \mathscr{H}_B = \mathscr{H}_N \otimes \mathscr{H}_A.$$

The register N is the normal qubit; A is the antiqubit. For both registers, the basis values 0 and 1 are called bulge and disk, respectively. They are finite labels only.

The sole unknown parameter is the normalized source weight $\theta \in [0, 1]$, defined by

$$\rho_\theta = \theta\,|00\rangle\langle 00| + (1-\theta)\,|11\rangle\langle 11|.$$

Thus θ is the relative weight of the first source component, not an absolute injection rate.

Assumption. The observer receives independent preparations of ρ_{θ} and does not receive the source-component label or a classical token containing the unknown value of θ. The preparation mechanism and any purification of ρ_{θ} belong to the supplier's interface.

This assumption prevents an expectation-value oracle from being hidden in the notation. If a simulator prepares from a known numerical value of θ, the same theorem is a sampling or validation theorem for that simulator; it is not acquisition of previously unknown Galactic information. This is the token-versus-acquisition distinction recorded in XIX-C [CheungXIXC].

Declared opposite response and relaxation

Fix, before collecting data,

$$0 < \varphi < \frac{\pi}{4}, \qquad 0 \le \gamma < 1.$$

These are known calibration inputs, not parameters estimated by the main inference map.

Let

$$Y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad V_\varphi = e^{-i\varphi Y} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}.$$

The declared local response is

$$W_\varphi = V_\varphi \otimes V_\varphi^{\dagger}.$$

The antiqubit designation means precisely the inverse factor $V_{\varphi}^{\dagger}$ in this equation. It is a linear unitary on an ordinary two-dimensional Hilbert space. No antiunitary transformation, unknown-unitary inversion algorithm, or reversal of physical time is asserted.

We include the fixed entangling gate

$$\mathrm{CZ} = \mathrm{diag}\,(1, 1, 1, -1), \qquad U_\varphi = \mathrm{CZ}\,W_\varphi.$$

The inverse-response claim concerns the two local factors of W_{φ}, not inversion of the complete interacting channel.

Put $h = 1 - \gamma$, and define

$$L_0 = \mathrm{kb}\,00 + \sqrt{h}\,\mathrm{kb}\,11, \qquad L_1 = \sqrt{\gamma}\,\mathrm{kb}\,01.$$

The one-step channel is

$$\Phi(X) = \sum_{\ell=0}^{1} H_\ell X H_\ell^\dagger, \qquad H_\ell = (I_N \otimes L_\ell) U_\varphi.$$

The coherent rotation is a two-zone mixing analog. The map *L_1* transfers the antiqubit basis value *1* to *0*. Because these basis values encode zones, this is a *declared relaxation surrogate*. It is not simultaneously an independently encoded energy-loss process. No vacuum, positronium spin, photon, or physical annihilation sector has been installed.

Lemma. The map *Φ: 𝒮(ℋ_B) → 𝒮(ℋ_B)* is completely positive and trace preserving.

Proof. The displayed Kraus form proves complete positivity. Moreover,

$$L_0^\dagger L_0 + L_1^\dagger L_1 = \mathrm{kb}\,00 + h\,\mathrm{kb}\,11 + \gamma\,\mathrm{kb}\,11 = I_A.$$

Since *U_{φ}* is unitary,

$$\sum_{\ell=0}^{1} H_\ell^\dagger H_\ell = U_\varphi^\dagger (I_N \otimes I_A) U_\varphi = I_B.$$

This proves trace preservation.

Access and observation boundary

The accessible data factor is $B = NA$, read only through its computational-basis occupation projectors

$$P_r = \mathrm{kb}\,rr, \qquad r \in \mathscr{R} := \{00, 01, 10, 11\}.$$

The record register C has basis $\{|r\rangle_C : r \in \mathscr{R}\}$, and its observable algebra is

$$\mathrm{Diag}(C) = \left\{ \sum_{r \in \mathscr{R}} c_r \mathrm{kb}\,rr_C : c_r \in \mathbb{C} \right\}.$$

An optional laboratory spectator S is untouched. An optional reference Q is also untouched and is included for complete channel comparisons, not as assistance to the observer. The accessible recorded-spectator algebra is

$$\mathrm{Diag}(C) \otimes \mathscr{L}(\mathscr{H}_S).$$

The supplier's source labels, channel environments, and measurement-device complements are outside this access boundary.

The newly returned object is C alone. An existing spectator may remain on a comparison interface, but neither N nor A is a quantum output. Allowed continuations may read C, act on S, and allocate fresh names. They may not reuse the consumed names of N or A.

The destructive instrument and its complete identity

Record and residual are specified together

For any retained factor T, including $T = S \otimes Q$, define

$$D_r = \langle r|_B \otimes I_T,$$

$$\mathscr{M}_B^T(X) = \sum_{r \in \mathscr{R}} \mathrm{kb}\,rr_C \otimes D_r X D_r^\dagger,$$

$$\mathscr{J}_B^T(X) = \sum_{r \in \mathscr{R}} \mathrm{kb}\,rr_C \otimes \mathrm{Tr}_B[(P_r \otimes I_T) X (P_r \otimes I_T)].$$

When T is one-dimensional, the T-blocks are scalars and the output is a classical record state.

The complete one-shot instrument is

$$\mathscr{I}^T = \mathscr{M}_B^T \circ (\Phi \otimes \mathrm{id}_T).$$

Its selective r-branch is the corresponding subnormalized block. All four outcomes are retained. There is no postselection.

Lemma. For every finite retained factor T,

$$\mathscr{M}_B^T = \mathscr{J}_B^T$$

as linear maps on all operators. The map $\mathcal{M}_B\hat{}T$ is CPTP, and

$$\mathrm{Tr}_C \mathcal{M}_B^T(X) = \mathrm{Tr}_B X.$$

Consequently $\mathcal{I}\hat{}T$ is CPTP and

$$\mathrm{Tr}_C \mathcal{I}^T(X) = \mathrm{Tr}_B X.$$

Proof. Let

$$A_r = |r\rangle_C \otimes D_r.$$

Then

$$\mathcal{M}_B^T(X) = \sum_r A_r X A_r^\dagger, \qquad \sum_r A_r^\dagger A_r = \sum_r P_r \otimes I_T = I_B \otimes I_T.$$

Thus the map is completely positive and trace preserving.

Expand an arbitrary operator as

$$X = \sum_{r,s \in \mathcal{R}} \mathrm{kb}\, rs_B \otimes X_{rs}.$$

Both the r-block of $\mathcal{M}_B\hat{}T(X)$ and the r-block of $\mathcal{J}_B\hat{}T(X)$ are X_rr. Hence the two maps agree on every operator, not merely on the source family.

Discarding the record gives $\sum_r X_rr = Tr_B\, X$, proving . Lemma proves that $\mathcal{I}\hat{}T$ is CPTP. Finally, any local trace-preserving map on B leaves the partial trace over B unchanged. In the present Kraus representation,

$$\mathrm{Tr}_B\,[(\Phi \otimes \mathrm{id}_T)(X)] = \mathrm{Tr}_B\left[\left(\sum_\ell H_\ell^\dagger H_\ell \otimes I_T\right) X\right] = \mathrm{Tr}_B X,$$

which proves .

The form of this proof matches the complete recorded-channel identity in source En10-A@1 [CheungRelational]; the proof above is local and does not import its clock assumptions.

Remark. For a state ρ_BT, the conditional T-state on outcome r is the normalized r-block when that block has positive trace. It may depend on r. Equation states that the *unconditional* T-marginal is unchanged. Conditional steering is not a failure of the complete identity. No normalized state is assigned to a zero-probability branch.

Linear consumption is an additional operational declaration

Definition. A quantum context is a finite set of pairwise distinct live quantum names. A unitary instruction acts on names in the context and preserves them. Allocation introduces a globally fresh name. The instruction *Enter(q)* removes q from the quantum context and returns one classical basis value. Subsequent instructions may use only live quantum names. Classical computation does not introduce a quantum name.

Lemma. Under Definition , destructive readout of both names in $B = NA$ has the interface

$$\mathrm{Q}_N \otimes \mathrm{Q}_A \otimes \mathrm{Q}_S \mathrm{multimap}\, \mathrm{Bit}^2 \otimes \mathrm{Q}_S.$$

No legal continuation uses the consumed N- or A-name as a quantum input.

Proof. The first Enter instruction removes its measured name. The second removes the other measured name. The resulting quantum context contains only the previously retained spectator names. Induction over any subsequent instruction sequence shows that unitary operations preserve this context, classical operations introduce no quantum names, and allocations introduce only globally fresh names. Neither consumed name can reappear.

Consumption is therefore not inferred from complete positivity alone. It follows from the declared name discipline and agrees with the instrument codomain. A physical implementation that retains a measured carrier but removes access to it may realize the same interface; it must not present that carrier as an unpriced quantum return.

The finite Enter theorem and estimation contract

Put

$$t = \sin^2\varphi, \qquad d = 1 - 2t = \cos(2\varphi), \qquad h = 1 - \gamma.$$

Under ,

$$0 < t < \frac{1}{2}, \qquad d > 0, \qquad h > 0.$$

Define the calibrated affine coefficients

$$a = \gamma + ht, \qquad b = hd > 0.$$

Assumption. A positive integer n is chosen before data collection. The n source preparations satisfy Assumption ; the one-shot device is repeated with independent fresh environments and without access to the hidden source labels. The reduced input on the data registers is $\rho_{\theta}^{\otimes n}$, and the ideal repeated record map is $\mathscr{I}^{\otimes n}$. There is no unaccounted adaptive stopping, record deletion, or selection of successful trials.

For the complete word $r_i = (x_i, y_i)$, define

$$Z_i = 1_{\{y_i = 0\}}, \qquad K = \sum_{i=1}^{n} Z_i, \qquad \overline{Z} = \frac{K}{n}.$$

The separate classical inference map returns

$$\hat{\theta} = \Pi_{[0,1]}\left(\frac{\overline{Z} - a}{b}\right), \qquad \Pi_{[0,1]}(u) = \max\{0, \min\{1, u\}\}.$$

For definiteness, this is a classical channel from word lists to integer labels $k \in \{0, \ldots, n\}$, with the declared interpretation

$$k \mathrm{longmapsto} \hat{\theta}_k = \Pi_{[0,1]}\left(\frac{k/n - a}{b}\right).$$

A CPTP matrix extension is

$$\mathscr{D}_n(X) = \sum_{\mathbf{r} \in \mathscr{R}^n} \langle \mathbf{r} \mathrm{rvertX} \, |\mathbf{r}\rangle \, |K(\mathbf{r})\rangle\langle K(\mathbf{r})|.$$

Its Kraus operators are $|K(r)\rangle\langle r|$, whose squared moduli sum to the identity. This map does not return a data qubit.

Theorem. Freeze the interface of Section , adopt Born semantics and Definition , and impose Assumption for the estimation statements. Then the following hold.

• **Complete instrument identity.** The instrument $\mathscr{I}$ is a complete CP instrument with classical output. For every finite untouched retained factor T, including an arbitrary reference,

$$\mathscr{I}^T = \mathscr{J}_B^T \circ (\Phi \otimes \mathrm{id}_T)$$

on all input operators. Discarding its record leaves the unconditional T-marginal unchanged.

• **Consumption and residual scope.** Both measured data names are consumed. The newly returned object is the classical word r; no quantum N- or A-output belongs to the instrument. A conditional spectator or inaccessible implementation complement is not a return of those names.

• **Declared likelihood.** The one-shot probabilities are

$$p_{00}(\theta) = \theta(1-t)(1-ht) + (1-\theta)t(\gamma + ht),$$
$$p_{01}(\theta) = ht(1-t),$$
$$p_{10}(\theta) = \theta t(1-ht) + (1-\theta)(1-t)(\gamma + ht),$$
$$p_{11}(\theta) = h[\theta t^2 + (1-\theta)(1-t)^2].$$

In particular, Z_i are independent Bernoulli variables with

$$q_\theta = \Pr_\theta\{Z_i = 1\} = a + b\theta.$$

• **Separate finite-shot inference.** The estimator is the maximum-likelihood estimator for the *coarsened Bernoulli data* $Z_1, \ldots, Z_n$. It is not asserted to be the maximum-likelihood estimator for the complete four-outcome record.

For every $\varepsilon > 0$,

$$\Pr_{\theta}\{|\widehat{\theta} - \theta| > \varepsilon\} \le 2e^{-2nb^2\varepsilon^2}.$$

For $0 < \alpha < 1$, define

$$R_n(\alpha) = \min\left\{1, \frac{1}{b}\sqrt{\frac{\log(2/\alpha)}{2n}}\right\}.$$

Uniformly over $\theta \in [0, 1]$,

$$\Pr_{\theta}\{|\widehat{\theta} - \theta| \le R_n(\alpha)\} \ge 1 - \alpha.$$

Thus a sufficient shot budget for a radius at most $0 < \varepsilon < 1$ is

$$n \ge \left\lceil \frac{\log(2/\alpha)}{2b^2\varepsilon^2} \right\rceil.$$

• **Fisher-information ledger.** For $0 < \theta < 1$, the achieved one-shot information in the coarsened data is

$$F_A(\theta) = \frac{b^2}{q_\theta(1 - q_\theta)}.$$

The complete word has information

$$F_C(\theta) = d^2\left[\frac{1}{p_{00}(\theta)} + \frac{\gamma^2}{p_{10}(\theta)} + \frac{h^2}{p_{11}(\theta)}\right].$$

They satisfy

$$F_A(\theta) \le F_C(\theta) \le B_\theta := \frac{1}{\theta(1 - \theta)}.$$

The last bound also applies to any POVM on the post-step data state $\Phi(\rho_{\theta})$. Independent repetition multiplies the respective Fisher informations by n.

The output interpretation is *finite-model estimate under declared access*. The theorem supplies neither a quantum-return decoder nor a physical identification with Galactic transport.

Proof. We prove each claim on the declared interface.

Instrument and consumption. Lemma proves the complete identity and its reference extensions. Equivalently, the r-branch on a record-only interface has Kraus rows

$$B_{r\ell} = \langle r|H_\ell.$$

Their completeness is

$$\sum_{r,\ell} B_{r\ell}^\dagger B_{r\ell} = \sum_{\ell} H_\ell^\dagger\Big(\sum_r P_r\Big)H_\ell = I_B.$$

Lemma proves the name statement.

Likelihood. The squared moduli of both V_{φ} and $V_{\varphi}^{\dagger}$ are the transition matrix

$$\begin{pmatrix} 1 - t & t \\ t & 1 - t \end{pmatrix}.$$

The diagonal gate CZ changes phases but not computational populations.

Conditional on source component 00, the normal output has probabilities $1 - t, t$. The antiqubit's probability of value 1, after relaxation, is ht. Conditional on source component 11, the normal output has probabilities $t, 1 - t$, and the antiqubit's probability of value 1 is $h(1 - t)$.

The diagonal output populations conditional on either source component factor accordingly. This remains true despite coherences in the quantum state: amplitude damping transfers diagonal populations as stated, and its off-diagonal matrix elements do not contribute to these computational probabilities. Mixing the two source components gives . Summing the outcomes with $y = 0$ gives

$$q_\theta = \gamma + h[t + (1 - 2t)\theta] = a + b\theta.$$

The product preparation and product device assumptions give independence.

Likelihood maximization. For a count $K = k$, the Bernoulli likelihood is proportional to

$$q^k(1-q)^{n-k}, \qquad q \in [a, a+b].$$

Its unconstrained maximizing value is k/n; restricting it to the interval gives the projection of k/n onto $[a, a+b]$. Applying the affine inverse gives , including endpoint counts.

Concentration. For completeness, let Z be Bernoulli with mean q, and set

$$g(u) = \log \mathbb{E} e^{u(Z-q)}.$$

Then $g(0) = g^{'}(0) = 0$, and $g^{''}(u)$ is the variance of a Bernoulli variable under an exponentially tilted law. Hence $g^{''}(u) \le 1/4$, and integration yields

$$g(u) \le \frac{u^2}{8} \qquad (u \in \mathbb{R}).$$

Independence and exponential Markov bounds therefore give

$$\Pr\{|\overline{Z} - q| > v\} \le 2e^{-2nv^2}.$$

This is the Bernoulli instance of Hoeffding's inequality [Hoeffding1963].

Projection onto $[0, 1]$ cannot increase distance from $\theta \in [0, 1]$. Consequently,

$$|\hat{\theta} - \theta| \le \left| \frac{\overline{Z} - a}{b} - \theta \right| = \frac{|\overline{Z} - q_\theta|}{b}.$$

Taking $v = b\varepsilon$ proves . Taking $v = \sqrt{(\log(2/\alpha)/(2n))}$, and using $|\theta - \theta| \le 1$, proves –. Equation follows directly.

Achieved Fisher information. For a Bernoulli law with derivative $q_{\theta}^{'} = b$,

$$F_A = \frac{b^2}{q_\theta} + \frac{b^2}{1-q_\theta} = \frac{b^2}{q_\theta(1-q_\theta)}.$$

Differentiating gives

$$(p_{00}^{'}, p_{01}^{'}, p_{10}^{'}, p_{11}^{'}) = (d, 0, -\gamma d, -hd),$$

which proves . All four probabilities are strictly positive under the calibration assumptions.

Information comparisons. Let J denote the hidden source component, with $Pr_{\theta}\{J = 0\} = \theta$. Its score is

$$s_\theta(J) = \begin{cases} 1/\theta, & J = 0, \\ -1/(1-\theta), & J = 1. \end{cases}$$

Thus

$$\mathbb{E}_\theta s_\theta(J) = 0, \qquad \mathbb{E}_\theta s_\theta(J)^2 = B_\theta.$$

For any outcome R obtained through a θ-independent conditional law given J, the outcome score equals

$$\partial_\theta \log p_\theta(R) = \mathbb{E}_\theta[s_\theta(J) \mid R].$$

Conditional Jensen gives

$$F_R = \mathbb{E}_\theta(\mathbb{E}_\theta[s_\theta(J) \mid R])^2 \le B_\theta.$$

This applies to the complete word C. It also applies to any POVM on $\Phi(\rho_{\theta})$: its probabilities are

$$\theta \operatorname{Tr}(E_j \Phi(|00\rangle\langle 00|)) + (1-\theta) \operatorname{Tr}(E_j \Phi(|11\rangle\langle 11|)),$$

which have exactly the required mixture form.

Finally, Z is a deterministic coarsening of C. Its score is the conditional expectation of the complete-word score given Z. A second application of conditional Jensen gives $F_A \le F_C$.

For independent repetitions, scores add and have mean zero, so the squared-score expectations add. This proves the n-shot information statements and completes the proof.

Remark. The input family has symmetric logarithmic derivative, on its support,

$$L_\theta^{\mathrm{src}} = \frac{1}{\theta}|00\rangle\langle 00| - \frac{1}{1-\theta}|11\rangle\langle 11|,$$

and therefore input quantum Fisher information

$$\mathrm{Tr}\,(\rho_\theta (L_\theta^{\mathrm{src}})^2) = B_\theta.$$

The preceding proof establishes the post-step measurement bound directly. Its relation to the standard optimization of classical Fisher information over measurements is consistent with the Braunstein–Caves framework [BraunsteinCaves1994]. No equality of *F_A*, *F_C*, and the post-step quantum Fisher information is asserted.

Remark. A confidence statement is a repeated-sampling coverage guarantee. It is not a posterior probability without additional Bayesian assumptions. The Fisher-information comparison is not by itself a finite-shot confidence theorem.

The unclipped estimator

$$\hat{\theta}^{\sim} = \frac{\overline{Z} - a}{b}$$

is unbiased and has variance

$$\mathrm{Var}_\theta\,(\hat{\theta}^{\sim}) = \frac{q_\theta(1-q_\theta)}{nb^2} = \frac{1}{nF_A(\theta)}.$$

Clipping may introduce bias, but decreases squared error relative to every $\theta \in [0, 1]$. Thus

$$\mathbb{E}_\theta(\hat{\theta} - \theta)^2 \le \frac{q_\theta(1-q_\theta)}{nb^2}.$$

No unbiased-estimator assertion is made for the clipped estimator.

An explicit rational instance

Freeze

$$\varphi = \frac{\pi}{6}, \qquad \gamma = \frac{1}{4}.$$

Then

$$t = \frac{1}{4}, \qquad h = \frac{3}{4}, \qquad d = \frac{1}{2}, \qquad a = \frac{7}{16}, \qquad b = \frac{3}{8}.$$

The complete probabilities become

$c\|ccccr$	00	01	10	11
hline $p_r(\theta)$	$\frac{7+32\theta}{64}$	$\frac{9}{64}$	$\frac{21-8\theta}{64}$	$\frac{27-24\theta}{64}$.

They are nonnegative and sum to one for every $\theta \in [0, 1]$.

The inference map is

$$\hat{\theta} = \Pi_{[0,1]}\left(\frac{16K/n - 7}{6}\right), \qquad R_n(\alpha) = \min\left\{1, \frac{8}{3}\sqrt{\frac{\log(2/\alpha)}{2n}}\right\}.$$

Its estimate values are rational at every finite *n*. A sufficient shot count is

$$n \ge \left\lceil \frac{32}{9\varepsilon^2}\log\frac{2}{\alpha} \right\rceil.$$

The information ledger is

$$F_A(\theta) = \frac{36}{(7+6\theta)(9-6\theta)},$$

and

$$F_C(\theta) = \frac{16}{7+32\theta} + \frac{1}{21-8\theta} + \frac{9}{27-24\theta}.$$

These are analytic formulas, not reported numerical execution.

Record-only recovery and the correct residual distinction

No unrestricted quantum return from the record alone

For compatible channels Λ, Γ, use half-diamond distance

$$d_\diamond(\Lambda,\Gamma) = \frac{1}{2}\|\Lambda - \Gamma\|_\diamond.$$

Proposition. Let $\mathscr{R}: \mathscr{L}(\mathscr{H}_C) \to \mathscr{L}(\mathscr{H}_B)$ be any CPTP map whose only input is the classical record. Then neither $\mathscr{R}\mathscr{M}_B$ nor $\mathscr{R}\mathscr{I}$ is the identity channel on an unrestricted four-dimensional input. In fact,

$$d_\diamond(\mathscr{R}\mathscr{M}_B, \mathrm{id}_B) \geq \frac{3}{4}, \qquad d_\diamond(\mathscr{R}\mathscr{I}, \mathrm{id}_B) \geq \frac{3}{4}.$$

Proof. On diagonal record inputs, any such map has the form

$$\mathscr{R}\Big(\sum_r p_r \mathrm{kb}\, rr_C\Big) = \sum_r p_r \sigma_r, \qquad \sigma_r = \mathscr{R}(\mathrm{kb}\, rr_C).$$

Consequently,

$$(\mathscr{R}\mathscr{I})(X) = \sum_r \mathrm{Tr}\,(E_r X)\sigma_r, \qquad E_r = \sum_\ell H_\ell^\dagger P_r H_\ell.$$

The effects E_r are positive and sum to I_B. The same expression with $E_r = P_r$ describes $\mathscr{R}\mathscr{M}_B$.

Such a measure-and-prepare channel has separable output across an untouched-reference/output cut. This is the entanglement-breaking structure studied in [HorodeckiShorRuskai2003]; here it follows immediately from the displayed form.

Take a four-dimensional reference and

$$|\Omega_4\rangle = \frac{1}{2}\sum_{j=0}^{3} |j\rangle_Q |j\rangle_B.$$

For any product state $\tau_Q \otimes \sigma_B$,

$$\langle\Omega_4|(\tau_Q \otimes \sigma_B)|\Omega_4\rangle = \frac{1}{4}\mathrm{Tr}\,(\tau_Q^{\mathrm{T}}\sigma_B) \leq \frac{1}{4}.$$

The same bound holds for every separable state by convexity. The identity channel preserves $|\Omega_4\rangle\langle\Omega_4|$, whose overlap with itself is one. Testing its projector therefore gives trace distance at least $1 - 1/4 = 3/4$. This particular referenced input lower-bounds the half-diamond distance, proving both inequalities.

The proposition concerns unrestricted quantum-state recovery from the record *alone*. It does not rule out parameter estimation, recovery of a restricted classical algebra, or a different protocol with an additional quantum receiver, entangled assistance, or access to a complete environment.

Residual information is not physically annihilated

The instrument is a map on an observer's accessible interface. A dilation may retain information in its inaccessible complement. For example, the isometry

$$|r\rangle_B \mathrm{longmapsto}\, |r\rangle_C |r\rangle_F$$

followed by tracing out F realizes computational record readout. Before the partial trace, joint coherences can be present in CF. They are not available as a quantum output of the record-only channel.

Thus the statement ``The quantum names are consumed" must not be replaced by ``All quantum information has been destroyed everywhere."

Likewise, an unmeasured spectator may continue to evolve. For example, readout of B on $|00\rangle\langle 00|_B \otimes \rho_S$ leaves ρ_S unchanged, and a subsequently declared channel on S can act on it. The positive statement is about the absence of measured data names from the output context, not a universal prohibition on residual dynamics.

This correction preserves the record-versus-residual distinction in Landmark I and source En10-A@1 [CheungI,CheungRelational]. The static-record versus recovery comparison in Oai265-A is a documentary

parallel, not a premise of Proposition [CheungOai265].

No exact finite-shot source recovery

Proposition. At any fixed finite n, no classical inference procedure on the complete word list returns the exact value of θ with probability one for every $\theta \in [0, 1]$.

Proof. Under the calibration assumptions every one-shot outcome has strictly positive probability for every $\theta \in [0, 1]$. Hence every finite word list has positive probability under every such θ.

Suppose a possibly randomized inference procedure returned θ_0 almost surely when the parameter was θ_0. For each word list its conditional output law would then have to be concentrated at θ_0, since each list has positive probability. The same conditional output laws are used at any different parameter θ_1, so the procedure would still return θ_0 almost surely. It cannot also return θ_1 almost surely.

The finite-confidence contract is therefore the substantive positive result; it is not a disguised exact-recovery claim.

Implementation tolerance on the same interface

A finite-shot allowance

Proposition. Keep the nominal coefficients a, b, source family, and fixed sample size. Suppose shot i uses a classical-output CPTP map $\mathscr{I}_i$ on the same comparison interface and

$$d_{\diamond}(\widetilde{\mathscr{I}}_i, \mathscr{I}) \le \delta \qquad (i = 1, \ldots, n).$$

Assume the resulting shot records are independent; they need not be identically distributed. Apply the nominal inference map . Then

$$\Pr_{\theta}\left\{ |\widehat{\theta} - \theta| \le \min\left\{ 1, \frac{\delta + \sqrt{\log(2/\alpha)/(2n)}}{b} \right\} \right\} \ge 1 - \alpha.$$

Proof. Let $q_i = \mathbb{E}_{\theta} Z_i$ for the implemented shot. Trace-distance control of the output states bounds every event probability difference, so

$$|q_i - q_\theta| \le \delta.$$

Consequently

$$\left| \frac{1}{n}\sum_i q_i - q_\theta \right| \le \delta.$$

The bounded-independent-variable argument in the main proof applies without identical means and gives

$$\Pr\left\{ \left| \overline{Z} - \frac{1}{n}\sum_i q_i \right| > \sqrt{\frac{\log(2/\alpha)}{2n}} \right\} \le \alpha.$$

Combine these inequalities and apply the same projection argument as before.

For a target radius ε, the displayed certificate has a finite sufficient shot budget only when $b\varepsilon > \delta$, namely

$$n \ge \left\lceil \frac{\log(2/\alpha)}{2(b\varepsilon - \delta)^2} \right\rceil.$$

This is a limitation of this nominal certificate, not a universal impossibility theorem for alternative calibrated procedures.

If these assumptions hold conditional on calibration information and on an event G with $Pr(G^c) \le \beta$, the same conditional proof followed by integration gives unconditional coverage at least $1 - \alpha - \beta$. The conditional independence and bias bounds must be justified; they do not follow from naming a device ``calibrated''. This is the same statistical caution illustrated by the finite-confidence contracts in Oai265-B [CheungOai265].

Gate errors, repeated records, and exact causality

Lemma. For compatible CPTP maps $G_1, \ldots, G_m$ and implemented maps $\widetilde{G}_1, \ldots, \widetilde{G}_m$ on matching interfaces,

$$d_\diamond(\widetilde{G}_m\cdots\widetilde{G}_1, G_m\cdots G_1) \le \min\left\{1, \sum_{j=1}^{m} d_\diamond(\widetilde{G}_j, G_j)\right\}.$$

Proof. Subtract the two compositions and replace one factor at a time. Each term contains one difference of corresponding maps, composed before and after with channels. Diamond norm is contractive under those channel compositions. The triangle inequality gives the sum, and half-diamond distance between channels is at most one.

This proves a tolerance statement only after the comparison interfaces are fixed. It is the local counterpart of the resource-controlled complete-channel allowance used in En08-B [CheungEn08B].

For n independent repetitions, the complete word-list channel has the separate bound

$$d_\diamond\left(\mathrm{bigotimes}_{i=1}^{n}\widetilde{\mathscr{I}}_i, \mathscr{I}^{\otimes n}\right) \le \min\{1, n\delta\}.$$

This is not the same ledger as the per-shot mean-bias allowance δ/b in .

Nor does a small allowance prove exact non-signalling or exact geometric containment. For example, let $\mathscr{T}$ be the constant qubit-output channel and $\mathscr{F}$ the channel that measures the input in the computational basis and prepares the corresponding basis state. Then

$$\Lambda_\eta = (1-\eta)\mathscr{T} + \eta\mathscr{F}, \qquad d_\diamond(\Lambda_\eta, \mathscr{I}) \le \eta,$$

but for every $\eta > 0$ the outputs on inputs 0 and 1 can differ. Approximate channel agreement is not exact independence from an input.

A hardware failure alphabet, if needed, must be added to a common comparison interface with all attempted outcomes retained. Discarding failed trials without an explicit statistical adjustment does not satisfy Assumption .

Resource, estimation, and residual ledgers

An explicit finite dilation and schedule

A fresh environment qubit E, initially in $|0\rangle$, realizes the antiqubit relaxation by the isometry

$$|0\rangle_A|0\rangle_E \mathrm{longmapsto}\, |0\rangle_A|0\rangle_E,$$
$$|1\rangle_A|0\rangle_E \mathrm{longmapsto}\, \sqrt{h}\,|1\rangle_A|0\rangle_E + \sqrt{\gamma}\,|0\rangle_A|1\rangle_E.$$

An explicit unitary extension is obtained by applying:

• a rotation $R_y(2\arcsin\sqrt{(\gamma)})$ on E, controlled by $A = 1$;

• $CNOT_{E \to A}$. Here $R_y(\omega) = e^{-i\omega Y/2}$. The two formulas in follow directly.

Destructive Enter of E, with its outcome not exported, gives the two Kraus branches L_0, L_1 and consumes the environment's laboratory name. Destructive Enter of N, A then produces the exported word.

A complete one-shot schedule, after the supplier delivers the data registers, is:

• allocate E in $|0\rangle$;

• apply V_{φ} and $V_{\varphi}^{\dagger}$ in parallel;

• apply CZ;

• apply the controlled environment rotation;

• apply $CNOT_{E \to A}$;

• Enter E and forget its unexported outcome;

• Enter N, A and export both outcome bits. Forgetting the unexported classical value is an interface operation. No physical cyclic erasure cost is assigned to it.

Computational ledger

The following counts refer to the declared primitive library: single-qubit rotations, CZ, controlled rotations, CNOT, fresh allocation, and destructive Enter. They are not counts in a synthesized hardware gate set.

toprule Resource Declared quantity or boundary

midrule Data input per shot Two supplied qubits, N, A.

Fresh laboratory ancilla One environment qubit E per shot.

Peak live laboratory qubits $3 + s$, if S consists of s retained laboratory qubits.

Coherent operations per shot Two one-qubit rotations and three two-qubit operations.

Coherent gate depth At most four layers: parallel local responses, CZ, controlled rotation, CNOT.

Destructive readouts Three one-qubit Enter operations: one unexported environment outcome and two exported data outcomes.

Readout rounds Two rounds if the two final data readouts are parallel.

Quantum data output None. Previously retained spectators remain; they are not measured-data returns.

Shots The predeclared n, satisfying the selected confidence budget.

Sequential laboratory allocations n fresh environment names and $2n$ supplied data names.

Classical record Two exported bits per shot. A complete histogram contains four counts summing to n.

Classical inference storage The declared estimator needs the integer count K; the complete histogram may also be retained.

Supplier preparation External and not priced by the laboratory gate count. Any source purification is inaccessible and separately held.

External reference Included in correctness comparisons. If physically laboratory-held, its storage must be added to the laboratory ledger.

Primitive measurement apparatus Not microscopically constructed or thermodynamically priced. Enter is an admitted primitive of

Sequential reuse of laboratory capacity does not imply reuse of a consumed name or cost-free restoration of a physical device. No storage of all n quantum inputs at once is assumed.

Estimation ledger

The named parameter and known calibration inputs are

$$\text{estimated:}\theta, \qquad \text{known:}\varphi, \gamma, a, b.$$

The achieved and comparison quantities are

$$nF_A(\theta) \le nF_C(\theta) \le nB_\theta,$$

together with the finite-confidence radius $R_n(\alpha)$, or its implementation-adjusted counterpart.

The denominator b is indispensable. If the calibration were changed so that $b = 0$, the selected Bernoulli statistic would be independent of θ, and its inference contract would fail. This is a statement about that statistic; some other record factor could still contain information in a different interface.

In particular, $\gamma = 1$ would make the antiqubit readout constant, while the normal-qubit marginal could remain source-dependent. The main theorem avoids this degeneracy and does not conflate loss of one statistic with loss of every accessible parameter.

Residual and omitted-parameter ledger

toprule Quantity Status in the frozen interface

midrule Measured N, A quantum names Consumed; no quantum output port.

Conditional spectator state Retained only if explicitly supplied; not determined by the record alone on arbitrary correlated inputs.

Source-component label Not supplied to the observer.

Source purification Supplier-side complement; inaccessible.

Relaxation and readout complements Outside record-only access; not declared annihilated.

Exact turbulence spectrum Absent from the preparation family and channel.

Galactic magnetic-field direction $\hat{n}$ Absent. The matrix Y is a prescribed control generator, not an acquired

Galactic field direction.

Absolute Galactic normalization Absent. Fixed trial count is not a clock-calibrated source rate.

Dimensionful diffusion length or coefficient Not estimated. No length scale, time step, or continuum transport equation is installed.

Positronium singlet probability Not measured: these qubits encode position labels, not two independent physical spin factors.

Annihilation, escape, or photon counts No corresponding output flags or sectors are installed.

Physical energy, heat, work, and reset cost No complete thermodynamic implementation is supplied.

bottomrule

What the record does not identify

Omitted nuisance parameters have zero information

Proposition. Let ζ denote any additional parameter that is absent from ρ_{θ}, Φ, the readout, and the declared repeated-shot model. Then the complete record law is independent of ζ. Where derivatives are defined, every classical Fisher-information entry involving ζ is zero.

No inference procedure on these records distinguishes two values of ζ with different output distributions.

Proof. The complete record probability is

$$\Pr_{\theta}\{r_1, \ldots, r_n\} = \prod_{i=1}^{n} p_{r_i}(\theta),$$

which contains no ζ. Its ζ-derivative is zero. A classical post-processing of identical input laws has identical output laws.

Thus absence of access is not asserted merely by verbal decree. For the omitted quantities listed in the ledger it is witnessed by their absence from the likelihood.

Conversely, if an unknown field direction, loss law, turbulence parameter, or calibration uncertainty is inserted into the channel, it may affect the records indirectly. Its identifiability and nuisance-parameter information must then be analyzed in the enlarged model. The present theorem cannot declare it inaccessible while silently using its influence.

The nominal scale $5 \times 10^{43}\ s^{-1}$ mentioned in the motivating plan is not a parameter of this construction. No value of an absolute Galactic rate, including that benchmark, is derived.

These occupation records do not certify inversion

Proposition. Replace $V_{\varphi}^{\dagger}$ on A in by V_{φ}, keeping the same source family, diagonal CZ, relaxation channel, and computational readout. The complete record law on every ρ_{θ} is unchanged.

Accordingly, the records used by the main theorem do not certify the sign of the antiqubit response.

Proof. The two rotations have identical squared matrix elements. The source family is diagonal. The computational populations after the local rotations therefore agree. The diagonal CZ leaves those populations unchanged, and amplitude damping transforms them by the same population rule. Computational readout consequently gives the same .

This equality is asserted on the declared source family. It is not an equality of the two channels on all coherent, reference-entangled input operators. A protocol that validates unitary inversion must add suitable coherent probes and its own comparison contract.

The construction is not empty of quantum coherence: before relaxation, for example, the amplitude matrix of $U_{\varphi}|00\rangle$ is

$$\begin{pmatrix} \cos^2\varphi & -\cos\varphi\sin\varphi \\ \cos\varphi\sin\varphi & \sin^2\varphi \end{pmatrix},$$

whose determinant is $2\cos^2\varphi\sin^2\varphi \neq 0$. That branch is entangled. Nevertheless the selected occupation

likelihood has an exact classical sampling description: sample the hidden source component and then sample the two conditional population transitions derived in the proof. No computational or metrological quantum advantage for this source-weight task is claimed.

A channel is not a thermal machine

Proposition. The visible channel Φ, the destructive record channel, and their complete reference identities do not determine a unique physical energy transfer, bath heat, work cost, or cyclic memory-reset cost.

Proof. In the dilation , assign the environment Hamiltonian

$$H_E^{(\varepsilon)} = \varepsilon \mathrm{kb}\, 11_E, \qquad \varepsilon > 0.$$

The reduced channel on the data is unchanged as ε varies. If the pre-relaxation data state has antiqubit population *p_1* in value *1*, the environment's energy increase is

$$\Delta E_E^{(\varepsilon)} = \varepsilon \gamma p_1.$$

Hence the visible channel alone fixes neither that energy transfer nor the Hamiltonian needed to interpret it. A pure blank environment has not thereby been identified as a thermal reservoir.

Even after chosen Hamiltonians are fixed, an independent unobserved process can alter a designated reservoir ledger without altering the data or record channel. For example, take a fuel qubit initially excited and a Gibbs reservoir qubit, both with gap ε. At inverse temperature β, the reservoir excitation probability is

$$r = \frac{1}{1 + e^{\beta\varepsilon}}.$$

Swapping the two qubits increases the designated reservoir energy by $\varepsilon(1 - r)$, while acting trivially on the visible interface. Repeating this independent process changes that ledger without changing the visible channel.

These examples consume blanks or fuel. They are not cyclic erasure protocols unless those resources and controls are also restored. They establish underdetermination, not a violation of a thermal inequality.

The proposition is a local version of the channel-versus-ledger separation in En07-B [CheungEn07B]. Landauer-type statements require a further physical setup, including the reservoir, initial correlations, Hamiltonians, process, and restoration requirements; a rigorous finite-reservoir treatment is given by Reeb and Wolf [ReebWolf2014].

Finite approximation is not a continuum identification

Any later larger-graph or continuum comparison must specify, at minimum:

• the target state and channel domains;

• encoding and decoding maps;

• the output and residual comparison interface;

• the error topology and a proved error bound;

• an actual resource schedule;

• independently supplied spatial and temporal calibration;

• a bridge connecting the named finite observables to the physical target. Lemma can price a finite approximation once these interfaces exist. It does not create them.

This maintains the separation between scheduled ancestry, operational influence, and independently supplied causal geometry in VII-J, and the finite-data cautions of En08-A and XIX-C [CheungVIIJ,CheungEn08A,CheungXIXC]. An observation boundary defined by available operations is not a derived causal cone.

Relationship to the supplied manuscript programme

The following comparisons identify methodological relations; they do not transfer hypotheses between models.

toprule Source Relationship to the present result

midrule Landmark I, version 2 Consumption and Born/residual characterization within its specified class. The present name rule and instrument are defined and proved locally.

Source En10-A@1 Template for complete destructive readout with subnormalized residual blocks and arbitrary references. Its clock-conditioned residual demonstrates why ``no residual can evolve'' would be too strong.

XIX-C, version 1 Distinction between a supplied token and an acquisition instrument, and between finite determinacy and continuum evolution.

En08-B, version 1 Complete-channel approximation and resource-ledger pattern after compatible comparison interfaces are supplied.

En08-A and VII-J Finite schedule, operational influence, and geometric comparison remain distinct.

Oai265-A and Oai265-B Static records do not imply quantum recovery; finite-confidence certificates require explicit complete-record statistical assumptions.

En07-B, version 1 Channel equality alone does not determine a thermal cost. Proposition supplies the local witness here.

XXII-B Candidate D, as named in the plan A proposed comparison label, not an imported theorem. Exact identity and tolerance statements used here are proved locally.

Inherited characteristic-graph programme No characteristic distribution, quantum gravitational coefficient, interacting differential, or Ward identity is constructed by this finite instrument.

bottomrule

The supplied XIV-J and XV-B excerpts concern separate analytic domains and unresolved graph-specific constructions [CheungAnalytic]. A four-dimensional instrument does not construct those missing products or supply their output-space admission. This document does not open the downstream XV-B exit, discharge XVI-N or Route B, or alter any version-specific O.1 verdict. In particular, a scoped later subsector result is not silently replaced by an older global label.

OpenAI catalogue entries remain documentary comparisons. They supply neither Φ, the source-access assumption, the Enter instrument, nor the statistical contract. No catalogue family is verified here.

Conclusion and verification boundary

The construction has the explicit chain

$\rho_\theta$xrightarrow Φ $\Phi(\rho_\theta)$xrightarrow $\mathscr{M}_B$ classical occupation wordxrightarrow $\mathscr{D}_n$ finite-confidence source-weight estimate.

Every arrow has a declared domain and output. The second arrow consumes the measured names. The third is classical post-processing and is not part of the readout primitive.

The philosophical discipline is therefore realized rather than merely repeated:

• complete instrument equality is proved, not inferred from agreement on one preparation;

• the classical record and conditional residual are kept separate;

• an estimator is provided with a sensitivity and shot ledger;

• omitted physical parameters are not recovered by relabeling finite outputs;

• thermodynamic and geometric conclusions are withheld because their interfaces have not been constructed.

This is a finite analog with an estimation contract. It is not a solution of the Galactic positron problem. Increasing the number of qubits or combining this subroutine with a classical transport code would require a new declared interface and would not automatically strengthen the conclusions proved here.

Verification boundary. The source uses standard pdfLaTeX packages and contains its bibliography internally. The algebraic and statistical arguments are supplied in the document. No successful pdfLaTeX execution, proof-assistant verification, experimental implementation, numerical benchmark, hardware compilation, independent referee review, or journal acceptance of this manuscript is asserted.

References

N. Cheung, ``The Enter Calculus: A Linear Quantum Lambda Calculus with an Operational Characterization of Measurement,'' *Frontiers in Physics* **14** (2026), article 1954943. doi:10.3389/fphy.2026.1954943. The publisher record identifies an accepted article and a forthcoming final formatted version. Its stated strategy class is not enlarged here.

N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, EntPhil-v2.tex, version 2. Enter Calculus Companion, introductory essay. Philosophical map, not a registered theorem or a proof premise.

N. Cheung, *Landmark I*, I.tex, version 2. Supplied manuscript excerpts, especially thm:main: characterization of destructive Enter within the specified strategy class. Cited with the scope of the supplied text; no independent verification of the complete manuscript is asserted.

N. Cheung, *En10-A*, En10-A.tex, version 1. Supplied manuscript excerpts, especially thm:main, eq:exact-channel-identity, eq:drop-record, and eq:linear-interface. Finite relational clock readout by destructive Enter. Called ``source En10-A@1'' in the present document.

N. Cheung, *XIX-C*, XIX-C.tex, version 1. Supplied manuscript excerpts, especially thm:main and the token/acquisition comparison. Finite incidence, complete-channel semantics, and continuum non-determination.

N. Cheung, *En08-B*, En08-B.tex, version 1. Supplied manuscript excerpts, especially thm:main and lem:telescoping. Resource-controlled operational approximation on a prescribed comparison interface.

N. Cheung, *En08-A*, En08-A.tex, version 1. Supplied manuscript excerpts, especially thm:main. Scheduler consistency and conditional geometric representation are distinct claims.

N. Cheung, *VII-J*, VII-J.tex, version 1. Supplied manuscript excerpts. Scheduled ancestry, operational influence, and independently supplied causal geometry remain separate.

N. Cheung, *Oai265-A*, Oai265-A.tex, version 1, especially thm:separation; and *Oai265-B*, Oai265-B-v2.tex, version 2, especially thm:main. Supplied manuscript excerpts. Record/recovery separation and complete-record finite-confidence certification; neither supplies an astrophysical identification.

N. Cheung, *En07-B*, En07-B.tex, version 1. Supplied manuscript excerpts, especially prop:no-landauer-from-channel. Thermodynamic underdetermination from observable-channel equality alone.

N. Cheung, *XIV-J*, XIV-J.tex, version 1, especially thm:J2 and cor:noadmission; and *XV-B*, XV-B.tex, version 2, especially thm:release. Supplied manuscript excerpts. Separate reconstruction and characteristic-map programme; documentary comparison only.

X. Song, S. S. Borjigin, F. Salvati, Y.-X. Wang, N. Yunger Halpern, D. R. M. Arvidsson-Shukur, and K. Murch, *Superconducting antiqubits achieve optimal phase estimation via unitary inversion*, arXiv:2506.04315 (2025). doi:10.48550/arXiv.2506.04315. Platform-specific inverse response and a separate qubit–antiqubit metrology protocol.

W. Hoeffding, ``Probability inequalities for sums of bounded random variables,'' *Journal of the American Statistical Association* **58**, no. 301 (1963), 13–30. doi:10.1080/01621459.1963.10500830.

S. L. Braunstein and C. M. Caves, ``Statistical distance and the geometry of quantum states,'' *Physical Review Letters* **72** (1994), 3439–3443. doi:10.1103/PhysRevLett.72.3439.

M. Horodecki, P. W. Shor, and M. B. Ruskai, ``Entanglement breaking channels,'' *Reviews in Mathematical Physics* **15**, no. 6 (2003), 629–641. doi:10.1142/S0129055X03001709. arXiv:quant-ph/0302031.

D. Reeb and M. M. Wolf, ``An improved Landauer principle with finite-size corrections,'' *New Journal of Physics* **16** (2014), 103011. doi:10.1088/1367-2630/16/10/103011. arXiv:1306.4352.

T. Siegert, R. Diehl, G. Khachatryan, M. G. H. Krause, F. Guglielmetti, J. Greiner, A. W. Strong, and X. Zhang, ``Gamma-ray spectroscopy of positron annihilation in the Milky Way,'' *Astronomy & Astrophysics* **586** (2016), A84. doi:10.1051/0004-6361/201527510. arXiv:1512.00325. Astrophysical motivation only; no fit, rate, or spatial model is imported into the finite theorem.

Exact non-identification statement

The output is a **finite-model estimate under declared access**. The instrument does not recover a Galactic quantum state, does not identify its observation boundary with a light cone or a Galactic radius, and does not identify its records with the interstellar medium, the observed 511keV morphology, or the Galactic positron rate. No thermal machine, Landauer cost, energy ledger, spacetime emergence, Einstein dynamics, Page curve, or quantum evolution without an external time is implied. No inherited gravitational quantum coefficient is admitted, and no compilation is asserted. All stronger identifications require additional constructions that are not supplied here.