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En09-A Realization of Relational Time by Destructive Enter on a Clock Register

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Abstract

We construct a finite operational realization of Page–Wootters clock conditioning using destructive Enter on a named clock register. The complete output consists of a classical clock code and the surviving system; the measured clock names are consumed. An explicit channel identity proves equality of the Born record law and the unnormalized conditional system states with their Page–Wootters counterparts, including arbitrary untouched references. Under a separately stated covariance condition for the clock basis, stationarity of the joint preparation implies unitary relational evolution at the calibrated readings. An exact finite spectral construction establishes that these hypotheses are jointly inhabited. The probability and update formulas have two distinct justifications: they follow algebraically from the specified instrument, and, when the full Enter characterization hypotheses are satisfied, they are the measurement semantics selected by the previously established operational characterization. We do not rederive that characterization from abbreviated axiom names. We also give a typed hybrid propagator, complete-channel stability bounds, and refinement independence within an explicitly matched class. The philosophy of Enter is expressed as an interface discipline: a destructive record is not a surviving clock, a decoder, a thermodynamic ledger, or a spacetime geometry. The construction realizes relational extraction, not clock-free quantum gravity, a dynamical causal cone, or spacetime emergence.

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Introduction and exact scope

The Page–Wootters proposal describes system dynamics through correlations with an internal clock in a stationary joint description [PageWootters]. Finite clock history states and their circuit realizations provide a concrete setting in which this conditioning can be studied [BoetteRossignoli,McClean,Diaz]. The present manuscript supplies a particular operational extraction interface for such correlations: destructive Enter on the clock.

Enter is not an alternative name for an arbitrary measurement. Its defining access discipline is that the measured quantum names do not survive as quantum outputs. The readout of those names is classical. Unmeasured system names can nevertheless survive in the same complete process. This distinction is central to Cheung's Enter calculus and to the accompanying philosophy of destructive readout and refusal of identification [CheungEnter,CheungI,CheungPhilosophy].

Our claim is therefore an interface realization, not an identification of all relational-time formalisms with a

programming primitive. Mathematical conditioning on a clock projector need not itself consume a clock. The construction below chooses a destructive implementation, proves its complete channel, and accounts for the resource that this implementation consumes.

Three claims that must remain separate.

• *Extraction:* a clock-conditioned state is returned as the residual system accompanying a classical record.

• *Dynamics:* those conditional states satisfy a specified evolution law. This requires clock covariance in addition to stationarity.

• *Foundational semantics:* the measurement rule is selected by an operational characterization inside the full strategy class of the Enter theorem. The first is an explicit channel identity. The second is proved under stated mathematical hypotheses. The third invokes an identified predecessor result and is not inferred merely by writing down a Born-form instrument.

Non-claims installed at the outset. No theorem below constructs a residual matter clock, returns the consumed clock as an unknown quantum state, produces an exact all-real reading register, derives a metric or Einstein dynamics, or supplies a Page curve. External calibration, preparation, scheduling, and any subsequently supplied physical embedding remain declared inputs. In particular, finite operational consistency does not determine a Lorentz cone [CheungEn08A].

The manuscript is independent of the unresolved characteristic quantum-product programme described in the supplied XIV–XVI records. A finite readout cannot supply an absent field-theoretic coefficient, its representation, or its admission.

Frozen native data and measurement semantics

Registers, names, and interfaces

Fix disjoint finite sets of quantum names C and S, with

$$m = |C| \geq 1, \qquad n = |S|, \qquad d = 2^m.$$

Their Hilbert spaces are

$$\mathscr{H}_C = (\mathbb{C}^2)^{\otimes m}, \qquad \mathscr{H}_S = (\mathbb{C}^2)^{\otimes n}.$$

A register Q denotes an arbitrary finite untouched reference. It may be entangled with $C \otimes S$. It is not available to the clock-reading program.

Write

$$\mathrm{Z} = \{0, \ldots, d-1\}, \qquad \mathscr{H}_R = \mathbb{C}^d.$$

The output record algebra is the diagonal algebra in the distinguished basis $\{|k\rangle_R\}_{k \in Z}$. Thus a recorded output has the form

$$\Omega_{RSQ} = \sum_{k \in \mathrm{Z}} |k\rangle\langle k|_R \otimes \sigma_{SQ,k}, \qquad \sigma_{SQ,k} \geq 0, \qquad \sum_k \mathrm{Tr}\, \sigma_{SQ,k} = 1.$$

This is a classical–quantum state, not a coherent quantum return of the clock.

The frozen operational interfaces are:

• a preparation of ρ_{CS}, or a declared preparation channel with this output;

• the accessible clock input C;

• the surviving system output S;

• the retained classical record R;

• all untouched references, acted on by id_Q. If a purification, auxiliary token, encoding apparatus, or fresh clock supply is used, its ownership and access are declared separately. Nothing in the readout theorem acquires a state-dependent expectation token.

Clock basis and calibration

Choose an orthonormal clock basis

$$\{|\tau_k\rangle_C : k \in \mathrm{Z}\}, \qquad P_k = |\tau_k\rangle\langle\tau_k|_C.$$

Choose an injective calibration

$$c : \mathrm{Z} \to \mathbb{R}, \qquad c(k) = t_k, \qquad t_0 = 0.$$

The stored object is the finite code *k*. The real number *t_k* is its declared interpretation; no finite register is asserted to contain every real number exactly.

Let *B_C* be the calibrated basis-change unitary satisfying

$$B_C|\tau_k\rangle_C = |k\rangle_C.$$

Reading the clock means applying *B_C*, then destructive computational-basis Enter to every name in *C*. If the clock basis is already the computational basis, *B_C = I_C*.

The logical clock Hamiltonian need not be diagonal in the computational presentation. Indeed, confusing an energy basis with a time-reading basis can destroy the desired dynamical interpretation; see Example .

Stationarity and the stronger constraint

Fix self-adjoint finite matrices *H_C* and *H_S*, and put

$$K = H_C \otimes I_S + I_C \otimes H_S.$$

A density operator is *stationary* when

$$= 0.$$

A stronger zero-constraint preparation satisfies

$$K\rho_{CS} = 0.$$

For a positive density operator, means that its support lies in *ker K*, and implies . For a pure preparation the stronger condition is $K|\Psi\rangle = 0$.

These two conditions are not silently identified. The extraction identity requires neither. Stationarity is used only in the relational-evolution clause.

Definition. The calibrated clock basis is covariant for the readings *{t_k}* when

$$P_k = U_C(t_k)P_0U_C(t_k)^\dagger, \qquad U_C(t) = e^{-iH_Ct/\hbar}.$$

This is an explicit compatibility condition between the clock generator, reading basis, and external calibration.

Covariant clock observables are an established ingredient of relational quantum dynamics [Trinity]. Here we require only the finite sampled condition ; we do not import an infinite-dimensional physical-inner-product or Dirac-observable construction.

The imported Enter characterization

The distinction between a specified instrument and a derived measurement semantics is important enough to make explicit.

Hypothesis. Whenever a realization is described as using *derived Enter semantics*, its measurement strategy belongs to the full structural class of Cheung's characterization theorem and satisfies its composability, no-signaling, nullity, continuity, and typed record requirements in their stated forms.

The imported conclusion is that, for a pure finite configuration $|\Xi\rangle$, a measured subset, and outcome *k*, the destructive component $D_k|\Xi\rangle$ has probability

$$p_k = \|D_k|\Xi\rangle\|^2,$$

and, if $p_k > 0$, residual ray

$$[D_k|\Xi\rangle/\sqrt{p_k}\,].$$

No residual is prescribed on a zero-probability branch.

This is the conclusion of Landmark I, version 2, thm:main, with its stated strategy class [CheungI]; Cheung's journal article distinguishes specified Born semantics from their subsequent operational characterization [CheungEnter]. Hypothesis retains the full predecessor hypotheses: state sufficiency, coherent dynamics, evaluation conventions, finite configurations, and the precise record condition are not replaced by informal slogans.

We use the characterization as a cited semantic result, not as a new theorem proved from the five abbreviated requirement names. An arbitrary laboratory apparatus is not certified to belong to that class by this manuscript.

Lemma. Under Hypothesis , destructive readout in the clock basis has unnormalized branch map

$$\mathscr{I}_k(X) = D_k X D_k^\dagger, \qquad D_k = \langle \tau_k|_C \otimes I_S.$$

The same formula holds with every untouched reference.

Proof. For a pure input, multiplication of the normalized positive-branch residual projector by its probability gives

$$\|D_k|\Xi\rangle\|^2 \frac{D_k|\Xi\rangle\langle\Xi \mathrm{rvert} D_k^\dagger}{\|D_k|\Xi\rangle\|^2} = D_k|\Xi\rangle\langle\Xi \mathrm{rvert} D_k^\dagger.$$

A zero component gives the zero operator, without assigning a normalized residual.

For a mixed input, purify it with a reference *E*, apply the pure formula with *D_k ⊗ I_E*, and take the required reduced view. The result is *D_kXD_k^{†}*. This expression depends only on *X*, so it is independent of the purification or ensemble presentation. Keeping rather than tracing a reference gives the same formula with its identity factor.

Thus the density-operator channel used below is a consequence of the imported semantics when Hypothesis applies. Independently, it is an explicitly specified finite quantum instrument.

The complete relational Enter theorem

The instrument and the conditioning target

Define the complete destructive clock-readout channel by

$$\mathscr{M}_C(X) = \sum_{k \in \mathrm{Z}} |k\rangle\langle k|_R \otimes D_k X D_k^\dagger.$$

Equivalently, in the computational presentation,

$$\mathscr{M}_C(X) = \sum_k |k\rangle\langle k|_R \otimes (\langle k|_C \otimes I_S)(B_C \otimes I_S) X (B_C^\dagger \otimes I_S)(|k\rangle_C \otimes I_S).$$

For a preparation *ρ_CS*, define

$$\sigma_{S,k} = \mathrm{Tr}_C\,[(P_k \otimes I_S)\rho_{CS}(P_k \otimes I_S)],$$
$$p_k = \mathrm{Tr}\,\sigma_{S,k},$$
$$\rho_S(t_k) = \frac{\sigma_{S,k}}{p_k} \quad \text{when} p_k > 0.$$

Equation is the clock-conditioned Page–Wootters state for this finite observable.

The appropriate *channel* target is not the normalized quotient in . It is the linear recorded map

$$\mathscr{J}_C(X) = \sum_k |k\rangle\langle k|_R \otimes \mathrm{Tr}_C\,[(P_k \otimes I_S) X (P_k \otimes I_S)].$$

Keeping subnormalized blocks prevents postselection from being mistaken for a deterministic channel.

Theorem. For the frozen data of Section , the following statements hold.

• **Complete instrument identity.** The map *𝒩_C* is completely positive and trace preserving, and

$$\mathscr{M}_C = \mathscr{J}_C$$

as linear maps on all clock–system operators. This equality remains valid after tensoring with *id_Q* for every untouched reference.

• **Record law and residual.** On input *ρ_CS*, the probability of code *k*, hence of calibrated reading *t_k*, is

$$p_k = \mathrm{Tr}\,[(P_k \otimes I_S)\rho_{CS}].$$

On a positive branch, the surviving system is exactly *ρ_S(t_k)* of . Under Hypothesis , these are the probabilities and updates selected by the Enter characterization, not an additional measurement postulate introduced here.

• **Linear consumption.** The readout consumes every measured quantum name in *C*. Its quantum output context contains *S*, not *C*. A continuation in the same process cannot use a consumed clock name as a quantum input.

• **Relational evolution under covariance.** If *ρ_CS* satisfies and the reading basis satisfies , then

$$\sigma_{S,k} = U_S(t_k)\sigma_{S,0}U_S(t_k)^\dagger, \qquad U_S(t) = e^{-iH_S t/\hbar}.$$

In this complete covariant basis, *p_k = 1/d*, and therefore

$$\rho_S(t_k) = U_S(t_k)\rho_S(0)U_S(t_k)^\dagger.$$

• **Complete operational realization.** The record, surviving system, and reference extension realize precisely the declared relational extraction. Discarding the record yields

$$\mathrm{Tr}_R\,[(\mathscr{M}_C \otimes \mathrm{id}_Q)(X)] = \mathrm{Tr}_C X.$$

No quantum clock output or spacetime identification is part of this channel.

Proof. Define Kraus operators from *ℋ_C ⊗ ℋ_S* to *ℋ_R ⊗ ℋ_S* by

$$A_k = |k\rangle_R \otimes D_k.$$

Then *ℳ_C(X) = ∑_k A_kXA_k^{†}*, which is completely positive. Orthonormality of the clock basis gives

$$\sum_k A_k^\dagger A_k = \sum_k P_k \otimes I_S = I_C \otimes I_S.$$

Hence *ℳ_C* is trace preserving.

For an arbitrary operator *X*, expand in the clock basis:

$$X = \sum_{j,\ell} |\tau_j\rangle\langle\tau_\ell|_C \otimes X_{j\ell}.$$

Both

$$D_k X D_k^\dagger \quad \text{and} \quad \mathrm{Tr}_C\,[(P_k \otimes I_S)X(P_k \otimes I_S)]$$

equal *X_kk*. This proves on all operators, not only on the frozen preparation. Replacing *I_S* by *I_S ⊗ I_Q* proves the reference statement.

The *R*-marginal is

$$\mathrm{Tr}_S \mathscr{M}_C(\rho_{CS}) = \sum_k p_k |k\rangle\langle k|_R.$$

Cyclicity of the full trace and *P_k^2 = P_k* give

$$p_k = \mathrm{Tr}\,[(P_k \otimes I_S)\rho_{CS}(P_k \otimes I_S)] = \mathrm{Tr}\,[(P_k \otimes I_S)\rho_{CS}],$$

proving . Dividing the positive *k*-block by its trace gives exactly . Lemma supplies the semantic attribution under Hypothesis .

For consumption, the register-level readout has the first-order interface

$$\mathrm{Q}^{\otimes m} \otimes \mathrm{Q}^{\otimes n} \mathrm{multimap}\, \mathrm{Bit}^{\otimes m} \otimes \mathrm{Q}^{\otimes n}.$$

It is implemented by the calibrated basis change and destructive Enter on the *m* clock names. Each elementary readout removes its measured name from the quantum context. After all *m* readouts, the surviving quantum context is *S*. Tensor permutation and classical packaging do not recreate a quantum name. This proves the linear-resource assertion. The system output in is unmeasured residual output; it is not a quantum return of any name in *C*.

To prove the dynamical clause, fix $t = t_k$, write $U_C = U_C(t)$, $U_S = U_S(t)$, and use stationarity:

$$(U_C \otimes U_S)\rho_{CS}(U_C^\dagger \otimes U_S^\dagger) = \rho_{CS}.$$

Consequently,

$$(U_C^\dagger \otimes I_S)\rho_{CS}(U_C \otimes I_S) = (I_C \otimes U_S)\rho_{CS}(I_C \otimes U_S^\dagger).$$

For partial trace over C, the two-projector expression in equals the corresponding single-projector expression. Using covariance and unitary invariance of the partial trace,

$$\begin{aligned}\sigma_{S,k} &= \mathrm{Tr}_C\,[(U_C P_0 U_C^\dagger \otimes I_S)\rho_{CS}] \\ &= \mathrm{Tr}_C\,[(P_0 \otimes I_S)(U_C^\dagger \otimes I_S)\rho_{CS}(U_C \otimes I_S)] \\ &= U_S \mathrm{Tr}_C\,[(P_0 \otimes I_S)\rho_{CS}]U_S^\dagger \\ &= U_S \sigma_{S,0} U_S^\dagger.\end{aligned}$$

Thus all p_k coincide. Their sum is one and there are d outcomes, so $p_k = 1/d$. Normalization proves .

Finally, the clock-basis expansion gives

$$\mathrm{Tr}_R \mathscr{M}_C(X) = \sum_k X_{kk} = \mathrm{Tr}_C X.$$

The same calculation includes Q. The complete channel therefore returns exactly the record and residual specified above. Its codomain has no quantum C factor, and no geometric object occurs in its definition.

Remark. The operation acts as id_Q, but the *conditional* reference state may change when a clock outcome is selected. Such steering is compatible with : the unconditional reference marginal is unchanged.

The relational-evolution assertion on $S \otimes Q$ additionally requires stationarity of the full preparation under $K \otimes I_Q$. Stationarity of the CS-marginal alone does not imply stationarity of every extension.

Remark. The channel identity holds for every preparation and every calibrated orthonormal basis. Stationarity alone does not make an arbitrary basis a time observable. Covariance is the extra hypothesis that justifies the Schrödinger-type interpretation in .

Example. Take one clock qubit and one system qubit, with

$$H_C = -\hbar\omega|1\rangle\langle 1|, \qquad H_S = \hbar\omega|1\rangle\langle 1|, \qquad |\Psi\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}, \qquad \omega > 0.$$

Then $K|\Psi\rangle = 0$. Enter in the clock energy basis produces system states $|0\rangle$ and $|1\rangle$, each with probability $1/2$. However, $U_S(t)kb\ 00U_S(t)^{\dagger} = kb\ 00$ for every t; it cannot turn the first conditional state into the second.

The problem is not failure of the extraction identity. It is failure of the proposed basis to satisfy clock covariance for two distinct readings. Changing to the clock basis $|\tau_0\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$, $|\tau_1\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$, with $t_1 = \pi/\omega$, produces the conditional system states $|+\rangle$ and $|-\rangle$, which do satisfy the asserted evolution.

An exact finite stationary realization

The theorem's covariance and constraint assumptions are compatible, but not automatic. We now construct a nonempty exact class.

Proposition. Let $d = 2^m$, $\omega > 0$, and let $\{\Pi_a\}_{a=0}^{d-1}$ be mutually orthogonal projections on $\mathscr{H}_S$ with $\sum_a \Pi_a = I_S$; zero projections are allowed. Set

$$H_S = \hbar\omega \sum_{a=0}^{d-1} a\Pi_a.$$

Choose a clock energy basis $\{|e_a\rangle\}_{a=0}^{d-1}$ and define

$$H_C = -\hbar\omega \sum_{a=0}^{d-1} a|e_a\rangle\langle e_a|,$$

$$\Delta = \frac{2\pi}{d\omega}, \qquad t_k = k\Delta,$$

$$|\tau_k\rangle = \frac{1}{\sqrt{d}} \sum_{a=0}^{d-1} e^{2\pi i a k/d} |e_a\rangle,$$

$$W|\varphi\rangle = \sum_{a=0}^{d-1} |e_a\rangle \otimes \Pi_a |\varphi\rangle.$$

Then $W: \mathscr{H}_S \to \mathscr{H}_C \otimes \mathscr{H}_S$ is an isometry, $KW = 0$, and the clock basis is covariant at the readings t_k. Moreover,

$$W = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |\tau_k\rangle \otimes U_S(t_k),$$

and, for every operator X,

$$\mathscr{M}_C(WXW^\dagger) = \frac{1}{d} \sum_{k=0}^{d-1} |k\rangle\langle k|_R \otimes U_S(t_k) X U_S(t_k)^\dagger.$$

All identities hold with every reference.

Proof. Orthogonality and completeness of the projections imply

$$W^\dagger W = \sum_a \Pi_a = I_S.$$

On the term $|e_a\rangle \otimes \Pi_a|\varphi\rangle$, the clock and system energies sum to zero, so $KW = 0$.

The finite Fourier identity

$$\frac{1}{d} \sum_{a=0}^{d-1} e^{2\pi i a(k-\ell)/d} = \delta_{k\ell}$$

proves orthonormality of $\{|\tau_k\rangle\}$. The sign in gives

$$U_C(t_k)|\tau_0\rangle = |\tau_k\rangle,$$

hence covariance.

Expanding the right side of , using

$$U_S(t_k) = \sum_a e^{-2\pi i a k/d} \Pi_a,$$

and summing over k, leaves precisely . Applying D_k to gives $D_k W = d^{-1/2} U_S(t_k)$. Substitution in proves . Tensoring the operator identities with an identity proves the reference versions.

Remark. This construction is exact for the stated commensurate spectrum. It does not encode an arbitrary prescribed Hamiltonian into the same finite additive constraint without an additional approximation or spectral-matching theorem. Its readings are periodic with period $2\pi/\omega$; an absolute winding number would require an additional classical interface.

The negative clock generator in is a mathematical constraint convention, not a thermodynamic claim. One may shift its spectrum upward and replace $K = 0$ by a fixed total-energy constraint without changing the covariance or readout calculation.

Proposition. For nonzero finite-dimensional $\mathscr{H}_C$, no matrices T_C, H_C satisfy

$$[T_C, H_C] = i\hbar I_C.$$

Proof. The trace of a matrix commutator is zero, whereas the trace of $i\hbar I_C$ is $i\hbar \dim \mathscr{H}_C \neq 0$.

This obstruction does not contradict Proposition . Sampled covariance of a finite periodic reading basis is not an exact canonical commutation relation.

Quantum-computing analogs and selective extensions

Circuit history states

History-state algorithms are established quantum-computing constructions [McClean,BoetteRossignoli,Diaz]. Their connection to Enter can be expressed without presuming that every history state satisfies the additive

constraint .

Corollary. Let *V_0, …, V_{d − 1}* be unitaries on $\mathscr{H}_S$, and define

$$W_V = \frac{1}{\sqrt{d}} \sum_{k=0}^{d-1} |k\rangle_C \otimes V_k.$$

Then *W_V* is an isometry, and computational-basis Enter on the clock realizes the channel

$$X \longmapsto \frac{1}{d} \sum_{k=0}^{d-1} |k\rangle\langle k|_R \otimes V_k X V_k^\dagger.$$

The clock names are consumed and the system names survive.

Proof. Orthogonality of the clock basis gives

$$W_V^\dagger W_V = \frac{1}{d} \sum_k V_k^\dagger V_k = I_S.$$

The *k*-branch destructive component is

$$(\langle k| \otimes I_S) W_V = d^{-1/2} V_k.$$

Equation follows from Theorem . Its resource assertion is the same typing calculation.

A preparation interface for *W_V* consists of

$$|+\rangle^{\otimes m} \otimes |\varphi\rangle \quad \longmapsto \quad \Big(\sum_k |k\rangle\langle k|_C \otimes V_k \Big) (|+\rangle^{\otimes m} \otimes |\varphi\rangle).$$

The controlled unitaries are declared operations. Exact synthesis from a particular finite gate library, its approximation error, and its storage cost are separate implementation obligations.

If *V_{k + 1} = G_kV_k*, define maps

$$L_k = \frac{1}{\sqrt{2}} (\langle k+1| \otimes I_S - \langle k| \otimes G_k), \qquad 0 \le k < d - 1.$$

Then the positive propagation Hamiltonian

$$H_{\text{prop}} = \sum_{k=0}^{d-2} L_k^\dagger L_k$$

annihilates *W_V*, because *L_kW_V = 0*. This is a legitimate history-state constraint, but generally not the noninteracting sum *H_C + H_S*. A propagation constraint is not silently substituted for the frozen additive constraint.

The grounded comparison

Coarse records, failure flags, and local POVMs

Proposition. Let *q: Z → Y* be a declared finite coarse-graining. Reporting only *y = q(k)*, while retaining *S*, gives branch maps

$$\mathscr{I}_y^q(X) = \sum_{k \,:\, q(k) = y} D_k X D_k^\dagger.$$

Their sum is trace preserving. On a positive branch,

$$\rho_S(y) = \frac{\sum_{k \,:\, q(k) = y} p_k \rho_S(t_k)}{\sum_{k \,:\, q(k) = y} p_k}.$$

This is generally a mixture of fine conditional states, not the state at a single exact time.

Proof. Classical relabeling adds the corresponding diagonal output blocks. Completeness follows by summing over all *y*. The normalized formula is division of the resulting subnormalized block by its trace.

Failure or rejection is treated in the same way: its flag is part of the complete output alphabet. If the rejected residual is discarded or replaced, that action is an additional declared channel. A successful normalized branch alone is not the complete experiment.

For a local clock instrument with Kraus operators *{F_{y, a}: $\mathscr{H}$_C → $\mathscr{H}$_{C^{'}}}_a*, suppose the entire output

C^{'} is also discarded or consumed, while *S* is untouched. Its residual system branch is

$$\sum_a \mathrm{Tr}_{C'}\left[(F_{y,a}\otimes I_S)X(F_{y,a}^\dagger\otimes I_S)\right] = \mathrm{Tr}_C\left[(E_y\otimes I_S)X\right], \quad E_y = \sum_a F_{y,a}^\dagger F_{y,a}.$$

The equality follows by pairing both sides with every system observable and using trace cyclicity. Thus a represented local POVM can provide a further destructive conditioning interface.

Equation does not say that effects determine arbitrary residual instruments. It uses the special assumptions that only the clock is acted on and that all quantum clock output is removed. Measuring an ancilla pointer while returning a quantum clock is a different interface.

The supplied X-Enter selective-readout theorem is an appropriate comparison for retaining failure branches and proving selective-map, rather than effect-only, agreement [CheungXEnter]. Its encoded phase model is not a premise or a specialization of the clock Hamiltonian construction here.

The philosophy of Enter as an interface discipline

The philosophical contribution is not an extra dynamical axiom. It is the requirement that an interpretation remain attached to the input, access rights, outputs, and continuation that actually support it [CheungPhilosophy]. For the present construction this discipline has five precise consequences.

A reading is a record-producing event

Before the readout, the preparation contains quantum correlations between a clock and a system. After the readout, the accessible interface contains a finite classical label and a conditional system. The correlations supply the relational description; Enter supplies the chosen operation that makes a classical label available.

The operation does not create the family *U_S(t_k)* from nothing. Its dynamical content was already fixed by the preparation, constraint, and clock calibration. Enter realizes extraction of that content.

Consumption is an access statement

A consumed quantum name is absent from the continuation's quantum context. This does not assert annihilation of the laboratory device, of its environment, or of every physical degree of freedom that implemented the channel.

A reversible dilation may retain information in inaccessible degrees of freedom. Such degrees of freedom are not thereby returned through the declared clock-reading interface.

Proposition. For $d \geq 2$, $\mathscr{M}_C$ has no channel left inverse on its full clock–system input space. In particular, with a fixed system preparation, its output cannot universally return an arbitrary unknown quantum clock state.

Proof. Let

$$|\chi_\pm\rangle = \frac{|\tau_0\rangle \pm |\tau_1\rangle}{\sqrt{2}},$$

and let *η_S* be any fixed system state. The distinct inputs $|\chi_{+}\rangle\langle\chi_{+}| \otimes \eta_S$ and $|\chi_{-}\rangle\langle\chi_{-}| \otimes \eta_S$ have the same output:

$$\frac{1}{2}(\mathrm{kb}\,00_R + \mathrm{kb}\,11_R)\otimes\eta_S.$$

A left inverse would have to map this single output to both inputs, which is impossible.

This does not exclude recovery of information from a separately declared code in the surviving system or another accessible factor. For example, a record-controlled inverse of recovers its encoded *system input*. That is not a return of the consumed clock names, and it does not invert $\mathscr{M}_C$ on the full input space.

The record is not the residual

The classical marginal contains *{p_k}*. The complete output also contains the blocks *{σ_{S, k}}*. Agreement of the first does not imply agreement of the second.

Proposition. There are recorded channels with the same clock outcome law on every input and different conditional system states.

Proof. Let *S* contain a qubit, let *Z_S* be its Pauli *Z*, and compare

$$\mathscr{F}(X) = \mathscr{M}_C(X), \qquad \mathscr{G}(X) = \sum_k |k\rangle\langle k|_R \otimes Z_S D_k X D_k^\dagger Z_S.$$

Unitary conjugation preserves each block trace, so their record laws agree on every input. For input $|\tau_0\rangle\langle\tau_0| \otimes kb + +$, the conditional qubit output is $|+\rangle$ for $\mathscr{F}$ and $|-\rangle$ for $\mathscr{G}$. The residual states are orthogonal.

This elementary example expresses the same interface separation emphasized by VIII-A's residual-obstruction theorem and XIX-C's record–residual analysis [CheungVIII,CheungXIXC]. It explains why complete recorded channels, rather than record frequencies alone, appear in the stability statements below.

A stochastic clock reading is not an expectation oracle

Proposition. Let $\mathscr{S}$ be a convex state set. If an affine channel assigns each $\rho \in \mathscr{S}$ a classical distribution concentrated at one label $\ell(\rho)$, and distinct labels are distinguishable, then ℓ is constant.

Proof. If $\ell(\rho) \neq \ell(\sigma)$, affinity forces the classical output on $a\rho + (1 - a)\sigma$, $0 < a < 1$, to be

$$a\delta_{\ell(\rho)} + (1 - a)\delta_{\ell(\sigma)}.$$

This distribution is not concentrated at one label, a contradiction.

Accordingly, the record k is a sample from a law determined by the clock preparation. It is not an exact single-run label of an arbitrary unknown expectation value. Finite supplied tokens, repeated-preparation estimation, and approximate acquisition are different tasks, as stressed in XIX-C's prop:token and prop:affine [CheungXIXC].

Neither a heat ledger nor a spacetime metric follows

Equation is a channel equality. It does not assign measurement work, heat, reset cost, an energy budget, or environmental leakage. A thermodynamic implementation requires its own apparatus and resource specification.

Likewise, the code k, calibrated scalar t_k, and program schedule are not a metric tensor or a matter-defined relational lapse. Em02-E's classical lapse construction uses its own background and relational-map hypotheses [CheungEm02E]. No comparison map identifying that lapse with the present clock code has been constructed.

Thus the analog in Table is useful precisely because it does not promote a finite interface correspondence into an identity with gravitational spacetime.

Typed hybrid evolution and the two notions of time

The relational label t_k and the external operational schedule parameter s have different roles. The former labels a conditional state extracted from one preparation. The latter locates preparation, readout, and later operations in a declared experiment.

Live and recorded interfaces

Before readout, use the live operator space

$$\mathscr{A}_- = \mathscr{B}(\mathscr{H}_C \otimes \mathscr{H}_S).$$

After readout, use

$$\mathscr{A}_+ = \left\{ \sum_k |k\rangle\langle k|_R \otimes X_k : X_k \in \mathscr{B}(\mathscr{H}_S) \right\}.$$

The space $\mathscr{A}_\{+\}$ is a finite classical–quantum algebra. There is no surviving clock factor in it.

Choose a readout event with ordered germs

$$s_*^- < s_*^+.$$

They denote the interfaces immediately before and after the event. They may share a numerical scheduling coordinate with distinct event-order tags; no positive physical duration is inferred from this notation.

Let

$$\mathscr{U}_-(s, u) = \mathrm{Ad}_{e^{-iK(s-u)/\hbar}}$$

be the declared pre-event propagator. On a stationary preparation it leaves the preparation unchanged.

For the post-event interface, let $\mathscr{V}_k(s, u)$ be any declared CPTP propagator satisfying

$$\mathscr{V}_k(s,v)\mathscr{V}_k(v,u) = \mathscr{V}_k(s,u), \qquad \mathscr{V}_k(u,u) = \mathrm{id}.$$

Define

$$\mathscr{U}_+(s,u)\Big(\sum_k |k\rangle\langle k|_R \otimes X_k\Big) = \sum_k |k\rangle\langle k|_R \otimes \mathscr{V}_k(s,u)(X_k).$$

The minimal construction chooses every $\mathscr{V}_k = id$. Time-dependent Hamiltonians or other admitted post-event dynamics are also allowed. No time-homogeneous semigroup is assumed.

Proposition. Define the typed operational propagator by

$$T(s,u) = \begin{cases} \mathscr{U}_-(s,u), & u \le s < s_*^-, \\ \mathscr{U}_+(s,s_*^+)\mathscr{M}_C\mathscr{U}_-(s_*^-,u), & u \le s_*^- < s_*^+ \le s, \\ \mathscr{U}_+(s,u), & s_*^+ \le u \le s. \end{cases}$$

With the event germs included as endpoints, every expression is a CPTP map between its declared interfaces and

$$T(s,v)T(v,u) = T(s,u)$$

for every compatible ordered triple.

Proof. The pre-event and post-event maps have their stipulated composition laws. A crossing interval contains exactly one readout. Splitting such an interval before the readout composes two pre-event factors; splitting it after the readout composes two post-event factors. In either case the factors reduce to the middle line of . No composition introduces a second $\mathscr{M}_C$. Complete positivity and trace preservation are preserved under composition.

The type distinction is necessary. Writing $\mathscr{M}_C\mathscr{M}_C$ would be meaningless on these interfaces: the first readout has already consumed its clock input.

Relational transport is not a second clock measurement

Under the covariance hypotheses, the mathematical family

$$\mathscr{T}_{\mathrm{rel}}(t,u) = \mathrm{id}_R \otimes \mathrm{Ad}_{U_S(t-u)}$$

acts on the recorded interface and satisfies a composition law. It transports a conditional system state from the represented reading u to t while leaving the stored record unchanged. It does not create a new observation of the consumed clock or rewrite the original event as if it had occurred at a new reading.

Multiple physical clock readings require additional clock resources or a different nondestructive protocol, with their own complete instruments. One destructive readout of a history state is not, by itself, a multi-time intervention experiment.

Complete-channel stability and refinement independence

Reference-stable comparison

For compatible finite linear maps define

$$\|\Lambda\|_\diamond = \sup_{r \ge 1} \sup_{X \ne 0} \frac{\|(\Lambda \otimes \mathrm{id}_r)(X)\|_1}{\|X\|_1}, \qquad d_\diamond(\Phi,\Psi) = \frac{1}{2}\|\Phi - \Psi\|_\diamond.$$

For states, $d_tr(\rho,\sigma) = 1/2\|\rho - \sigma\|_1$. The half-diamond normalization gives $0 leq d_\{\diamond\}(\Phi,\Psi) \le 1$ for channels and includes arbitrary reference-entangled inputs [Watrous].

By ,

$$d_\diamond(\mathscr{M}_C, \mathscr{J}_C) = 0.$$

This is equality on the full fixed interface, not merely agreement at one calibrated preparation.

Proposition. Suppose $\mathscr{M}_C$ is a channel with the same classical record and system interface, with classical output blocks, and

$$d_\diamond(\widetilde{\mathscr{M}_C}, \mathscr{M}_C) \le \varepsilon.$$

For every reference-entangled input, write its two outputs as

$$\sum_k |k\rangle\langle k|_R \otimes \widetilde{\sigma}_{SQ,k}, \qquad \sum_k |k\rangle\langle k|_R \otimes \sigma_{SQ,k}.$$

Then

$$\frac{1}{2}\sum_k \left\|\widetilde{\sigma}_{SQ,k} - \sigma_{SQ,k}\right\|_1 \le \varepsilon, \qquad \frac{1}{2}\sum_k \left|\widetilde{p}_k - p_k\right| \le \varepsilon.$$

Every subsequent complete channel preserves the same error upper bound. If $p_k \ge p_{min} > 0$ and $\varepsilon < p_{min}$, then $\tilde{p}_k > 0$ and

$$d_{\mathrm{tr}}\left(\frac{\sigma_{SQ,k}}{p_k}, \frac{\widetilde{\sigma}_{SQ,k}}{\widetilde{p}_k}\right) \le \min\left\{1, \frac{\varepsilon}{p_{\min}}\right\}.$$

Proof. The diamond bound gives trace distance at most ε for the two complete outputs. Trace norm is additive on their orthogonal classical blocks, proving the first inequality in . Taking the classical marginal is a channel and proves the second. Channel postprocessing contracts trace distance, including with a reference.

Fix k, abbreviate

$$\sigma = \sigma_{SQ,k}, \quad \widetilde{\sigma} = \widetilde{\sigma}_{SQ,k}, \quad p = p_k, \quad q = \widetilde{p}_k, \quad a = \left\|\sigma - \widetilde{\sigma}\right\|_1.$$

The classical marginal bound implies $|p - q| \le \varepsilon$; thus $q > 0$ under the stated lower bound. The sum of the norms of all other output blocks is at least $|p - q|$, because their total trace difference is $q - p$. Therefore

$$a + |p - q| \le 2\varepsilon.$$

Using positivity and $Tr\ \tilde{\sigma} = q$,

$$\frac{1}{2}\left\|\frac{\sigma}{p} - \frac{\widetilde{\sigma}}{q}\right\|_1 \le \frac{a + |p - q|}{2p} \le \frac{\varepsilon}{p_{\min}}.$$

State trace distance is also at most one.

A positive lower probability bound is essential for a uniform normalized-branch statement. Complete-channel convergence alone does not control conditioning on arbitrarily rare outcomes. The exact covariant finite model has $p_{min} = 1/d$, but this bound need not remain uniform if the clock dimension grows.

Telescoping on compatible complete interfaces

Lemma. Let $G_1, \ldots, G_N$ and $\mathcal{G}_1, \ldots, \mathcal{G}_N$ be compatible channels, with corresponding factors on the same comparison interfaces. Then

$$d_\diamond(\widetilde{G}_N \cdots \widetilde{G}_1, G_N \cdots G_1) \le \min\left\{1, \sum_{j=1}^{N} d_\diamond(\widetilde{G}_j, G_j)\right\}.$$

Proof. Subtract the two compositions and replace one factor at a time:

$$\widetilde{G}_N \cdots \widetilde{G}_1 - G_N \cdots G_1 = \sum_{j=1}^{N} \widetilde{G}_N \cdots \widetilde{G}_{j+1}(\widetilde{G}_j - G_j) G_{j-1} \cdots G_1.$$

Channels have diamond norm one. Submultiplicativity therefore bounds each term by $\|\mathcal{G}_j - G_j\|_\diamond$. The triangle inequality and division by two prove the sum bound. The distance between channels is at most one.

This lemma applies to finite adaptive instruction trees after their classical controls and all reported outcomes are represented on the complete interfaces. A branch deleted by postselection is not such a trace-preserving factor.

The declared refinement class

Definition. Fix a complete finite target channel $\Phi: \mathcal{B}(\mathcal{H}_{in}) \to \mathcal{A}_{out}$, including its preparation, calibration, clock Enter instrument, all retained records, residual ownership, and any acceptance or rejection rule.

An admissible refinement supplies, for each scale j:

• an implemented channel Φ_j on these same external comparison interfaces;

• an ideal factor chain on compatible intermediate interfaces whose complete composition is exactly Φ;

• corresponding implemented factors, compared with those ideal factors on the same intermediate interfaces;

• an explicit assignment of factor errors to encoding, flow, event, classical record processing, and decoding;

• any needed padding, identification, or finite classical coarse-graining maps, included as factors rather than suppressed. Intermediate physical registers may depend on j. The external comparison algebra does not.

The existence of the ideal matched factor chains is an assumption of the refinement class, not a consequence of a shared finite diagram. In particular, this definition does not infer a continuum target from the native finite clock.

Theorem. In Definition , suppose the complete non-flow error totals are

$$\varepsilon_{\mathrm{enc},j}, \quad \varepsilon_{\mathrm{event},j}, \quad \varepsilon_{\mathrm{record},j}, \quad \varepsilon_{\mathrm{dec},j}.$$

Suppose the flow factors correspond to intervals of lengths $h_{j,\ell}$, with

$$\sum_{\ell} h_{j,\ell} \le L, \qquad \max_{\ell} h_{j,\ell} \le \delta_j,$$

and have half-diamond errors bounded by $Ah_{j,\ell}^{1+\alpha}$, where $A < \infty$ and $\alpha > 0$ are independent of j, ℓ. Then

$$d_{\diamond}(\widetilde{\Phi}_j, \Phi) \le \min\left\{1, \varepsilon_{\mathrm{enc},j} + AL\delta_j^{\alpha} + \varepsilon_{\mathrm{event},j} + \varepsilon_{\mathrm{record},j} + \varepsilon_{\mathrm{dec},j}\right\}.$$

If $\delta_j \to 0$ and the four non-flow totals tend to zero, the complete channels converge to Φ.

Any two admissible refinement families satisfying these conditions for the same frozen target converge to the same limit.

Proof. The total flow error satisfies

$$\sum_{\ell} Ah_{j,\ell}^{1+\alpha} \le A\delta_j^{\alpha} \sum_{\ell} h_{j,\ell} \le AL\delta_j^{\alpha}.$$

Apply Lemma to the complete matched factor chain and group the remaining factor errors by their declared roles. This proves .

For two families with bounds $b_j \to 0$ and $b_j^{'} \to 0$,

$$d_{\diamond}(\widetilde{\Phi}_j, \widetilde{\Phi}_j^{'}) \le d_{\diamond}(\widetilde{\Phi}_j, \Phi) + d_{\diamond}(\Phi, \widetilde{\Phi}_j^{'}) \le b_j + b_j^{'} \to 0.$$

Both families consequently have the same channel limit.

This is the precise error decomposition

$$\varepsilon_{\mathrm{enc}} + \varepsilon_{\mathrm{flow}} + \varepsilon_{\mathrm{event}} + \varepsilon_{\mathrm{record}} + \varepsilon_{\mathrm{dec}}.$$

A one-step error of order $O(h)$ alone does not force the accumulated error to vanish. Nor does a proposed product formula automatically supply the complete-instrument estimates required here. The supplied En05-A comparison distinguishes geometry, schedule, and channel semantics [CheungEn05A].

The XXI-A–C identifiers in the development plan motivate this hybrid and complete-experiment interface [CheungXXIOutline]. Their full texts were not supplied. The estimates used here are proved in this manuscript rather than attributed to unavailable proposition numbers.

Three convergence questions

Refinement must distinguish:

• convergence of externally calibrated classical readings;

• convergence of the quantum residual channel after forgetting some or all records;

• convergence of the complete recorded experiment on a common record–residual interface. The third implies the relevant marginal channel convergences. Neither the first nor the second, by itself, implies the third. Proposition illustrates the residual gap even when the record laws agree exactly.

Finite classical codes also do not converge to an exact all-real record in total variation merely because their calibrated values become dense. For example, let μ_j be any probability measure with finite support in $[0, 1]$, and let λ be uniform Lebesgue probability. If $F_j = \operatorname{supp} \mu_j$, then

$$\mu_j(F_j) = 1, \qquad \lambda(F_j) = 0.$$

Hence their total-variation distance is one at every scale. A weak limit can still exist, but it is a different convergence claim. Fixed finite bins or a specified bounded-statistic interface can be used instead; the binning or testing map must be declared.

Finally, common finite samples do not select a unique interpolation, generator, or continuum limit. XIX-C's prop:two-limits and prop:interpolation record precisely this non-determination [CheungXIXC]. Theorem avoids it by freezing the target and restricting the comparison class explicitly.

Optional causal embedding

No physical causal geometry is needed for the channel identity. If a realization is assigned locations, a causal embedding can be added as a separate contract.

Let the complete finite dependency graph include every actual quantum input, classical control, preparation, routing operation, and retained record. Assign each event a location p in a set with a supplied transitive causal relation preceq. Require every dependency edge $p \to q$ to satisfy $p \mathrel{\mathit{ppreceq}} q$.

Proposition. Under this edge condition, every dependency path lies in preceq. For local channel realizations of the graph, Heisenberg support of a terminal observable is contained in its scheduled ancestor region. Thus operational influence is bounded by that region and, through the certified embedding, by the supplied causal relation.

Proof. Path containment is transitivity. For support containment, pull the terminal observable backward through the circuit. The adjoint of a trace-preserving local channel sends an identity on an untouched factor to the identity, and can enlarge support only to the input factors of that event. Induction through the finite graph places the support in the ancestor region. An intervention on a factor outside that region cannot change the terminal reduced output, because its pulled-back observables act as the identity there.

Scheduled ancestry, actual operational influence, and preceq remain three different relations. Channels may fail to use a scheduled edge, while entanglement may produce correlations without intervention-induced signalling. The embedding contract does not reconstruct the supplied relation from the program.

These distinctions match VII-J's two targets: complete decoded simulation and physical embedding of the actual elementary operations [CheungVIIJ]. They do not derive a Lorentz-invariant speed or select a Lorentz cone; En08-A's scheduler-consistency separation remains applicable [CheungEn08A].

If a fixed geometric packet or a source-replacement controller is attached, its matching certificates, guards, and residual ownership must also be supplied. The VIII constructions are comparisons for such contracts, not a geometric conclusion of the clock-readout theorem [CheungVIII].

Exact release and standing boundaries

Corollary. For one declared finite clock–system preparation and one complete destructive clock-readout event:

• the retained classical code supplies the calibrated relational label;

• the surviving system block is exactly the Page–Wootters conditional state;

• the measured clock names are consumed;

• the complete recorded instrument agrees with its relational extraction target on every input and with every reference;

• sampled covariance and stationarity imply the stated conditional unitary evolution;

• under the full Enter characterization hypotheses, the Born record law and projective residual update have the inherited derived status;

• matched refinements obey the complete-channel budget and class-relative uniqueness theorem.

Proof. Items (i)–(vi) are Theorem , with Lemma for the semantic attribution. Proposition gives an exact inhabited finite constraint class. Item (vii) is Theorem .

What the release does not do. It does not remove external preparation or scheduling time. It does not construct an internal matter clock that remains available after the declared destructive event. It does not derive a dynamical

cone, a continuum metric, Einstein dynamics, a field-theoretic measurement apparatus, or fully quantized gravitational relational observables. A finite history state and its record are not automatically spacetime emergence.

The classical relational-observable and lapse results associated with VI and Em02-E retain their own equation spaces, backgrounds, and quantum obligations [CheungBoundaries,CheungEm02E]. No Enter readout closes those additional obligations.

Analytic non-transfer. No characteristic joint product, quantum coefficient, interacting Ward identity, or downstream admission is constructed. The XIV-I internal amplitude and XIV-J supported-sector reference are not replaced by a finite clock instrument. XV-B version 2 may exist as a technical continuation while the distinct downstream quantum-admission exit remains unopened. This manuscript does not change that status and does not discharge XVI-N [CheungBoundaries].

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is unchanged. These statements record scope; they are not an independent audit or a universal nonexistence theorem.

Conclusion. The philosophical and mathematical content can be summarized as

$$\text{boxed} \Rightarrow \begin{array}{l} \text{declared history correlations + calibrated destructive Enter} \\ \text{classical relational label + correct conditional system residual,} \\ \text{clock names consumed,} \qquad \mathscr{M}_C = \mathscr{J}_C, \qquad d_\diamond(\mathscr{M}_C, \mathscr{J}_C) = 0. \end{array}$$

The label is a record, not a surviving clock. The record is not a decoder. The channel is not a thermal machine. The analog is not an identification with spacetime.

appendix

Corrections to the proposed theorem architecture

The following corrections are necessary for the completed statement to be mathematically and operationally sound.

• **A clock basis must be declared.** Computational-basis Enter realizes the intended observable only when the clock basis is already computational or a calibrated basis change is included.

• **Stationarity does not imply the desired evolution for an arbitrary observable.** Sampled covariance is explicit in Definition ; Example supplies a counterexample without it.

• **Stationarity and a zero-energy constraint differ.** Equations and are not interchangeable. The exact finite construction satisfies the stronger condition.

• **Uniform readings are a consequence in the exact covariant complete basis.** One cannot simultaneously assume arbitrary clock weights and exact covariance of every basis projector for a stationary preparation.

• **Only the measured register's quantum output is absent.** The complete process still returns the unmeasured system. Saying that the entire process has ``only a classical output'' would incorrectly discard its residual.

• **The Born attribution is conditional on the full predecessor class.** Writing the instrument in Born form proves its algebraic law, not the predecessor's operational characterization. Hypothesis identifies the imported result and its applicability condition.

• **Normalized conditioning is not a linear channel.** The instrument uses subnormalized blocks. Zero-probability branches have no prescribed normalized state.

• **Reference stability is an operator statement.** Equality on one frozen state would not establish diamond equality. Equation holds on every operator. Relational evolution of a reference-inclusive preparation additionally requires its own full stationarity.

• **A consumed clock cannot be measured again in the same continuation.** The hybrid composition crosses the readout exactly once. Further physical readings need additional resources or a different instrument.

• **General circuit histories need not satisfy the additive constraint.** Their propagation Hamiltonian is kept distinct from $H_C + H_S$.

• **Refinement uniqueness is class-relative.** The ideal target, calibration, complete output interface, and local consistency estimates are frozen. Shared finite samples alone do not prove uniqueness.

• **No unsupplied theorem is silently imported.** The XXI-A–C material available here is an outline-level comparison. The hybrid and telescoping results needed for En09-A are stated and proved above.

Source provenance and verification status

The bibliography distinguishes journal articles, internal manuscript records, and outline-only material. Internal documents are cited by the supplied identifier and version rather than by an invented publication date, DOI, or formal title.

The only imported nonstandard semantic theorem used in the main argument is the Enter characterization identified in Hypothesis . Its full proof is not reconstructed here. The finite channel identity, covariance argument, spectral realization, hybrid composition, stability estimates, and class-relative refinement result have explicit proofs in this manuscript.

The public publisher record for [CheungEnter], consulted for bibliographic metadata, gives acceptance on 28 September 2026 and states that the final formatted article is forthcoming. This status is not converted into a publication date for any internal manuscript.

The supplied internal records support the stated comparisons and standing non-claims. They are not treated as independent machine verification, and similarities in notation do not transfer their mathematical hypotheses to En09-A.

No successful pdflatex execution, proof-assistant check, numerical execution, hardware experiment, independent referee review, or journal acceptance is asserted. The source is arranged as a single conventional pdflatex manuscript with its bibliography included.

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