En04-A Under the Enter Philosophy—Readout, Recovery, and Residual Response: Magic, Complete Complements, and the Limits of a Geometric Interpretation

Ngo Cheung

Abstract

We formulate the proposed connection between quantum magic and matter-responsive geometry at a finite, already-prepared quantum carrier interface. The complete output is an isometry V: C → R ⊗ B, and residual response means that its complete complementary channel is not constant on the logical code. Exact recovery from R is equivalent to absence of that response. The proposed unconditional implication from non-Clifford resources to residual response is false. We prove explicit counterexamples: a Clifford copying circuit has a nonconstant complement, whereas non-Clifford logical evolution and even a fixed syndrome containing bipartite non-local magic can coexist with a constant complement and exact logical recovery. We therefore separate magic, residual response, and physical geometric backreaction. For a specified controlled-rotation family, we construct a non-Clifford responding program and obtain the exact formulas δ_full(θ) = (sin θ)/2, ε_rec^R(θ) = (1 − cos θ)/2, 0 ≤ θ ≤ π/2. A general first-order residual-observable witness gives a quantitative sufficient condition for response under a declared deformation. A Pauli-compression criterion separately characterizes constant complements for Clifford encoders. Destructive Born–Lüders readout consumes measured quantum names and returns a classical interface. Readout-and-drop of an unobserved residual is exactly partial trace and does not further change the radiation channel. The four-Majorana recovery obstruction, finite-channel certificates, access-preserving endpoint continuity, and finite-proxy recurrence are retained with their proper scope. No metric, field-to-memory receiver, admitted analytic quantum coefficient, or gravitational backreaction theorem is constructed.

Bibliographic record

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The corrected logical structure.

ord resource ⇏ ?residual response,
ing ⇏ ?constant complete complement,
plement ⇔ ?exact whole-code recovery from R,
se ⇏ ?physical geometric backreaction.

$$Arespondingnon-Cliffordmodelcanbeprovedafteritscoupling, code, preparation, accesspartition, andresponsewitnessarefixed. Itcannotbeinferredmerelyfr$$

Introduction and the corrected problem

The operational question addressed here begins after a preparation has produced a finite quantum carrier. It concerns a complete encoding or evolution map, an accessible output, an inaccessible residual, a quantum decoder, and a separate destructive observation.

The proposed slogan

``Non-local magic enables geometry to respond to matter''

contains several logically distinct statements. In the present channel formulation, the relevant operational condition is non-constancy of the complete residual channel. However, neither ordinary non-Cliffordness nor a generic notion of non-local magic is equivalent to this condition. Moreover, an inaccessible quantum memory is not automatically a geometric degree of freedom.

Exact quantum correction and no-hiding supply the recovery criterion [KnillLaflamme,BraunsteinPati]. Magic belongs to a resource theory relative to a specified stabilizer structure [VeitchEtAl]. These are different mathematical structures.

A related but more specialized literature studies nontrivial area operators and perturbative matter–geometry correlations in holographic or skewed subsystem codes [Cao2024,CaoEtAl2026]. Its hypotheses, subsystem decompositions, and magic quantities are not identical to a generic channel $C \to RB$. Section explains the separation.

Relation to the Cheung manuscripts

Landmark I gives a Born-law and positive-branch residual characterization inside its declared strategy class [CheungI]. En03-A supplies the minimal linear interface and ownership discipline [CheungEn03A]. We define the finite fragment locally and use the supplied characterization only at its stated documentary level.

Hawk04-A supplies complete preservation, the global restore map, the exact complementary-channel criterion, the precise no-hiding conclusion, and the four-Majorana control [CheungHawk04A]. Hawk04-B distinguishes quantum return, destructive observation, complete apparatus outputs, and operational discard [CheungHawk04B]. The statements needed below are proved locally.

Hawk02-E supplies the regulated microscopic architecture and the separation of source energy from residual independence [CheungHawk02E]. The complete-process and apparatus disciplines of En02-B, En02-C, En02-E, and Em03-C are maintained [CheungEn02B,CheungEn02C,CheungEn02E,CheungEm03C]. Availability of a decoder, existence of a channel, and executable implementation under a resource budget remain distinct.

The XIV/XV/XVI and OpenAI catalogue comparisons are documentary separations only. They do not supply a premise for any channel, magic, or response theorem in this paper.

Four repairs needed before completing the plan

• **Magic is not leakage.** A non-Clifford gate can act entirely within the recoverable logical sector. Conversely, a Clifford interaction can copy a basis label to the residual.

• **The restore map does not move the diary into the residual.** It recovers the diary from the complete output RB. Residual-only recovery follows when the radiation channel is constant, not merely from global preservation.

• **Readout-and-drop is not permanent physical deletion.** It is a trace in the observer's channel semantics. Its complete unitary dilation retains omitted apparatus outputs.

• **Residual response is not yet geometric response.** One must identify actual geometric observables and show that they distinguish the declared matter inputs in the same physical model.

The completed result is consequently a corrected operational bridge with explicit witnesses and counterexamples, not a proof of the original unconditional slogan.

Complete channels and residual response

Let $C \simeq \mathbb{C}^d$, $d \geq 2$, be a finite logical code. Output Hilbert spaces are separable. Channels act normally on trace-class operators and are completely positive and trace preserving, abbreviated CPTP.

For an isometry

$$V : C \to R \otimes B,$$

set

$$\mathcal{V}(X) = VXV^\dagger, \qquad \mathcal{M}(X) = \mathrm{Tr}_B\, \mathcal{V}(X), \qquad \mathcal{C}(X) = \mathrm{Tr}_R\, \mathcal{V}(X).$$

The residual B includes every inaccessible output: preparation purifiers, retained memories, uncollected modes,

displaced carriers, and inaccessible apparatus factors.

Distances are

$$d_{\mathrm{tr}}(\rho,\sigma)=\frac{1}{2}\|\rho-\sigma\|_1, \qquad d_{\diamond}(\Phi,\Psi)=\frac{1}{2}\|\Phi-\Psi\|_{\diamond}.$$

All channel statements retain arbitrary untouched references. For finite input, standard diamond-norm and Choi conventions are as in [Watrous].

Define the replacement channel

$$\mathscr{T}_{\sigma}(X)=\mathrm{Tr}\,(X)\sigma$$

and the quantities

$$\varepsilon^{R}_{\mathrm{rec}}(V)=\inf_{\mathscr{D}\,:\,R\,\to\,C\ \mathrm{CPTP}} d_{\diamond}(\mathscr{D}\mathscr{N},\mathrm{id}_C),$$
$$\delta_{\mathrm{full}}(V)=\inf_{\mathrm{sigmad}} _\diamond(\mathscr{C},\mathscr{T}_{\sigma}).$$

Definition. The complete residual responds to the logical code if $\mathscr{C}$ is not a replacement channel. It is nonresponding if $\mathscr{C}=\mathscr{T}_\{\sigma\}$ for one fixed state σ.

This concerns variation with the logical input at a fixed encoding or fixed time. Dependence of a fixed replacement state $\sigma_\{\lambda\}$ on an external coupling parameter λ is not, by itself, logical-input response.

No ``rigid geometry'' is defined by Cliffordness. When convenient, we call a *proved nonresponding interface* an operationally rigid interface. This terminology carries no metric assertion.

Lemma. For isometries into a common output space,

$$d_{\diamond}(\mathscr{V}_1,\mathscr{V}_2)\le\|V_1-V_2\|.$$

The same allowance holds after a common partial trace. CPTP pre- and postprocessing contract half-diamond distance.

On fixed output spaces, recovery error and distance to the replacement-channel set are one-Lipschitz:

$$|\varepsilon_{\mathrm{rec}}(\mathscr{N}_1)-\varepsilon_{\mathrm{rec}}(\mathscr{N}_2)|\mathrm{led}_{\diamond}(\mathscr{N}_1,\mathscr{N}_2),$$
$$|\delta_{\mathrm{full}}(\mathscr{C}_1)-\delta_{\mathrm{full}}(\mathscr{C}_2)|\mathrm{led}_{\diamond}(\mathscr{C}_1,\mathscr{C}_2).$$

Proof. For normalized vectors u, v,

$$d_{\mathrm{tr}}(|u\rangle\langle u|,|v\rangle\langle v|)=\sqrt{1-|\langle u,v\rangle|^2}\le\|u-v\|.$$

Apply this to referenced isometric outputs, then use convexity and partial-trace contraction.

For each decoder, the triangle inequality changes its error by at most the channel comparison allowance. Taking infima in both directions proves recovery stability. The same argument applies to distance from the replacement-channel set.

Lemma. Put $a_d = 1 - d\^\{-2\}$. Then

$$\inf_{\mathrm{sigmad}} _\diamond(\mathrm{id}_C,\mathscr{T}_{\sigma})=a_d.$$

Every channel from C has recovery error at most a_d and leakage distance from replacement channels at most a_d.

Proof. For any replacement state, the Bell-input output is $I_Q/d\otimes\sigma$. Its overlap with the target Bell projector is $d\^\{-2\}$, giving the lower bound a_d.

The uniform twirl over the $d\^2$ Weyl unitaries is $\mathscr{T}_\{I_C/d\}$. Their Bell outputs are mutually orthogonal. The triangle inequality and that Bell input give the matching distance a_d.

A decoder which ignores its input and prepares I_C/d gives the recovery upper bound. For a channel Φ,

$$\Phi\mathscr{T}_{I_C/d}=\mathscr{T}_{\Phi(I_C/d)}.$$

Postprocessing contraction gives the leakage upper bound.

Preservation, exact recovery, and response witnesses

Theorem. Let $J: C \to H_in$ be an isometry, let $|\Omega\rangle$ be a fixed normalized ancilla, and let U be unitary on the complete input space. Then

$$V|\psi\rangle = U(J|\psi\rangle \otimes |\Omega\rangle)$$

is an isometry. Its complete channel has the CPTP left inverse

$$\mathscr{L}(Y) = V^\dagger Y V + \mathrm{Tr}\,((I - VV^\dagger)Y)\tau_C, \qquad \tau_C = I_C/d.$$

For every code–reference trace-class operator X,

$$\|(V \otimes I)X(V^\dagger \otimes I)\|_1 = \|X\|_1.$$

Proof. The preparation and evolution compose to an isometry. Both terms of are completely positive; their traces sum to $Tr\ Y$. The second term vanishes on the encoded range, and $\mathscr{L}(VXV\^{\dagger}) = X$.

An isometric embedding preserves nonzero singular values, including after tensoring with a reference identity.

The map $\mathscr{L}$ returns C from RB. It neither deletes the code nor asserts that B alone carries a recoverable copy. This is the restore structure in [CheungHawk04A].

Theorem. For $V: C \to R \otimes B$, the following are equivalent:

• one CPTP decoder on R exactly recovers the entire code;

• $\mathscr{C} = \mathscr{T}_\{\sigma\}$ for one residual state σ;

• for every code matrix unit $E_ij = |i\rangle\langle j|$,

$$\mathscr{C}(E_{ij}) = \delta_{ij}\sigma;$$

•

$$J_{\mathrm{n}}(\mathscr{C}) = \frac{I_Q}{d} \otimes \sigma;$$

• for every bounded residual observable O,

$$V^\dagger(I_R \otimes O)V \quad \text{is a scalar multiple of} I_C.$$

Here

$$|\Phi_d\rangle = \frac{1}{\sqrt{d}}\sum_i |i\rangle_Q|i\rangle_C, \qquad J_{\mathrm{n}}(\Phi) = (\mathrm{id}_Q \otimes \Phi)(|\Phi_d\rangle\langle\Phi_d|).$$

When these conditions hold, there exist a Hilbert space K, a state σ_K, and an isometry $A: C \otimes K \to R$ such that

$$\mathscr{N}(X) = A(X \otimes \sigma_K)A^\dagger.$$

An exact complete decoder is

$$\mathscr{D}(Y) = \mathrm{Tr}_K\,(A^\dagger Y A) + \mathrm{Tr}\,((I - AA^\dagger)Y)\tau_C.$$

Proof. Linearity and expansion of the Bell projector give equivalence of (ii)–(iv). Duality gives

$$\mathrm{Tr}\,(O\mathscr{C}(X)) = \mathrm{Tr}\,(V^\dagger(I \otimes O)V X).$$

Thus (ii) implies (v). Conversely, in (v), evaluate at τ_C to identify the scalar as $Tr\ (O\mathscr{C}(\tau_C))$. Since bounded observables separate trace-class operators, $\mathscr{C}(X) = Tr\ (X)\mathscr{C}(\tau_C)$, proving (ii).

Assume (ii), and diagonalize

$$\sigma = \sum_\alpha p_\alpha |b_\alpha\rangle\langle b_\alpha|, \qquad p_\alpha > 0.$$

The support condition permits the expansion

$$V|i\rangle = \sum_\alpha \sqrt{p_\alpha}\,|r_{i\alpha}\rangle|b_\alpha\rangle.$$

Constancy on every matrix unit gives

$$\langle r_{j\beta}, r_{i\alpha}\rangle = \delta_{ij}\delta_{\alpha\beta}.$$

Consequently $A(|i\rangle|\alpha\rangle) = |r_{i\alpha}\rangle$ is an isometry, and $\sigma_K = \sum_{\alpha} p_{\alpha}|\alpha\rangle\langle\alpha|$ gives . Formula is CPTP and recovers every matrix unit.

Conversely, dilate an exact decoder by an isometry $L: R \to C \otimes G$. Stinespring dilation is the standard channel structure theorem [Watrous,KSW]. The map $Z = (L \otimes I_B)V$ has

$$Z|i\rangle = |i\rangle \otimes |\eta_i\rangle,$$

because the decoded marginal is pure. Exact recovery of E_{ij} forces $\langle \eta_j, \eta_i\rangle = 1$. All η_i coincide. Tracing the decoded register and G therefore gives (iii).

These are the exact correction/no-hiding conditions [KnillLaflamme,BraunsteinPati] used in [CheungHawk04A]. The local proof additionally identifies the residual-observable test used below.

Corollary. If $\mathcal{N}$ is constant, B exactly recovers the code. If $\mathcal{C}$ is constant, R exactly recovers it. Both complementary outputs cannot independently recover the entire unknown code when $d \geq 2$.

Proof. Interchange R and B in Theorem . An exactly recoverable output has constant complement, which cannot itself recover two distinct inputs.

Lemma. For a finite input, $\delta_{full} = 0$ if and only if $\mathcal{C}$ is constant. If two code states satisfy

$$x = d_{\mathrm{tr}}(\mathcal{C}(\rho_0), \mathcal{C}(\rho_1)) > 0,$$

then

$$\delta_{\mathrm{full}} \geq x/2.$$

If $O = O^{\dagger}$, $\|O\| \leq 1$, and

$$w = |\mathrm{Tr}\,(O\mathcal{C}(\rho_0 - \rho_1))|,$$

then

$$\delta_{\mathrm{full}} \geq w/4.$$

Proof. For every replacement state, the triangle inequality gives

$$x \leq d_{\mathrm{tr}}(\mathcal{C}(\rho_0), \sigma) + d_{\mathrm{tr}}(\sigma, \mathcal{C}(\rho_1)) \leq 2d_{\diamond}(\mathcal{C}, \mathcal{T}_{\sigma}).$$

Also $w \leq \|\mathcal{C}(\rho_0 - \rho_1)\|_1 = 2x$.

If leakage is zero, choose σ_n with $d_{\diamond}(\mathcal{C}, \mathcal{T}_{\sigma_n}) \to 0$. Evaluation at τ_C gives $\sigma_n \to \mathcal{C}(\tau_C)$ in trace norm. Replacement-channel distance equals the corresponding state distance, so $\mathcal{C} = \mathcal{T}_{\mathcal{C}(\tau_C)}$.

Remark. A positive witness proves that the residual distinguishes some logical inputs. It does not prove residual-only recovery of the entire code. Joint recovery is guaranteed by the complete isometry; local recovery requires its own theorem.

Typed preparation, coherent programs, and Enter

The declared finite fragment

Types are

$$T ::= \mathrm{I} \mid \mathrm{Q} \mid \mathrm{Bit} \mid T \otimes T,$$

with observable algebras

$$\mathcal{A}_{\mathrm{I}} = \mathbb{C}, \qquad \mathcal{A}_{\mathrm{Q}} = M_2(\mathbb{C}), \qquad \mathcal{A}_{\mathrm{Bit}} = \mathbb{C} \oplus \mathbb{C}.$$

Classical registers are block diagonal. No conversion from a quantum register to a classical register is implicit.

A judgement $P: T \to T^{\prime}$ consumes the entire input interface and returns its declared output interface. Sequential and tensor composition require matching types. A live quantum name is not duplicated or silently omitted. This follows the minimal ownership reading of [CheungEn03A], rather than an unprovided larger calculus.

The primitives are pure allocation, specified finite unitaries, destructive computational readout, classical blockwise control, and classical discard:

$$\text{new} : \mathrm{I} \to \mathrm{Q}, \qquad \text{Enter} : \mathrm{Q}^{\otimes s} \to \mathrm{Bit}^{\otimes s}, \qquad \text{dropBit} : \mathrm{Bit} \to \mathrm{I}.$$

A specified unitary constant is a semantic object, not an assertion of exact finite-gate compilation, efficiency, or hardware realization.

For a measured subsystem S and surviving quantum subsystem T, write

$$M_x = \langle x|_S \otimes I_T.$$

The destructive readout instrument is

$$\mathscr{E}_S(\rho) = \sum_x |x\rangle\langle x|_K \otimes M_x \rho M_x^\dagger.$$

The measured names are consumed. For a pure input,

$$p_x = \|M_x\psi\|^2, \qquad \psi_x = M_x\psi/\sqrt{p_x} \quad (p_x > 0).$$

No normalized residual is assigned to a zero-probability branch.

These are the destructive Born–Lüders residual formulas supplied by Landmark I within its stated strategy class [CheungI]. We define the instrument directly; we do not reconstruct a measurement-uniqueness theorem from incomplete hypotheses.

Theorem. Every finite acyclic legal program denotes a CPTP hybrid-output channel and has path maps K_h satisfying

$$\sum_h K_h^\dagger K_h = I.$$

Reported outcomes group the unnormalized branch maps $X \mapsto K_hXK_h^{\dagger}$. The same statement holds with arbitrary untouched references. Every branch terminates within the finite height of its instruction tree.

Proof. Allocation and unitary gates are isometric. Readout satisfies $\sum_x M_x^{\dagger}M_x = I$. Classical discard sums blocks without conditional normalization.

Backward induction on the instruction tree proves completeness. Sequential composition composes the path maps; coarse-graining adds their CP maps. Tensoring each path map with a reference identity proves the complete-channel statement. A finite acyclic tree has finite maximum branch length.

This is the local version of the complete-history accounting in [CheungEn02B] and [CheungEn02C]. A selected success branch is not substituted for a deterministic decoder.

Complete coherent body and local quantum return

After a carrier dictionary exists, the coherent body has type

$$P_{\text{coh}} : Q_C \to Q_R \otimes Q_B$$

and form

a := new;?(C,a) := EncodeAndPrepare(C,a);?(R,B) := CoherentBody(C,a);?returnQ(R,B);

It contains no Enter.

A local decoder has type $P_{\{\mathscr{D}\}}: Q_R \to Q_C^{out}$. A finite CPTP decoder can be implemented semantically by

w := new;?(Cout,G) := DecoderUnitary(R,w);?y := Enter(G);?dropBit(y);?returnQ(Cout);

The Kraus isometry

$$L|\psi\rangle = \sum_j D_j|\psi\rangle|j\rangle_G$$

has a finite unitary extension after allocation. Readout and discard of G give $\sum_j D_jXD_j^{\dagger}$. The diary output remains quantum.

Applying this decoder locally to the coherent output yields a joint program returning (Cout,B). The observer controlling only Cout sees $\mathscr{DM}$. A radiation marginal is therefore a declared access restriction, not a rule silently deleting a live residual wire.

For non-power-of-two dimensions, all logical spaces are subspaces of padded qubit registers. Physical channels

are completed on invalid padded inputs, for example by fixed-state preparation. Such padding does not establish a physical field-to-qubit receiver.

Optional terminal observation

A prescribed terminal measurement may be appended after quantum return:

Cout := BasisUnitary(Cout);?x := Enter(Cout);?returnBit(x);

Its target is the measurement channel $\mathscr{M}$, not *id_C*. By contraction,

$$d_\diamond(\mathscr{D}\mathscr{N}, \mathrm{id}_C) \le \varepsilon \quad \Rightarrow \quad d_\diamond(\mathscr{M}\mathscr{D}\mathscr{N}, \mathscr{M}) \le \varepsilon.$$

The converse for one chosen measurement fails: computational dephasing preserves its computational record but not off-diagonal matrix units.

Enter is not a magic evaluator. A program may be mathematically analysed for nonstabilizerness, but the primitive semantics of *Enter* is the instrument .

Stabilizer structure and why the proposed implication fails

The resource convention must be fixed

Fix a qubit tensor decomposition and its Pauli frame. A Clifford unitary normalizes the Pauli group. A pure stabilizer state is a Clifford image of a computational basis state. The convex hull of such states is denoted *STAB*. A state outside this convex hull has stabilizer magic in this declared convention [VeitchEtAl].

The classification concerns operations and prepared states, not a promise that every arbitrary diary input is a stabilizer state. In particular, a Clifford program can process an unknown nonstabilizer diary.

Bosonic Gaussianity, fermionic Gaussianity, and qubit Cliffordness are not identified. An arbitrary qubit dictionary can preserve channel recovery while changing the Pauli-frame description. A magic claim must consequently fix its dictionary before evaluation.

Proposition. For

$$V' = (U_R \otimes U_B)V$$

with local output unitaries,

$$\delta_{\mathrm{full}}(V') = \delta_{\mathrm{full}}(V), \qquad \varepsilon^R_{\mathrm{rec}}(V') = \varepsilon^R_{\mathrm{rec}}(V).$$

These unitaries may be non-Clifford.

Proof. The residual channel is $\mathscr{C}\{'\} = Ad_U_B\ \mathscr{C}$. Replacement states are carried bijectively to their conjugates, and unitary channel distances are invariant. On radiation, a decoder may undo *U_R*, and conversely may prepend it when comparing the two tasks.

An output unitary mixing *R* and *B* is not covered: it changes the accessible task.

Clifford leakage and non-Clifford nonresponse

Proposition. There is a Clifford coherent program with

$$\delta_{\mathrm{full}} = \varepsilon^R_{\mathrm{rec}} = \frac{1}{2}.$$

There is a non-Clifford coherent program with

$$\delta_{\mathrm{full}} = \varepsilon^R_{\mathrm{rec}} = 0.$$

Proof. For the first example, allocate *B* in $|0\rangle$ and apply *CNOT_{C → B}*, returning the original carrier as *R*. Then

$$V_{\mathrm{copy}}|i\rangle = |i\rangle_R|i\rangle_B,$$

and both marginals are computational dephasing

$$\Delta_Z(\rho) = \frac{1}{2}(\rho + Z\rho Z).$$

The two residual basis outputs are orthogonal, giving *δ_full ≥ 1/2*. The replacement *I_2/2* gives the matching

upper bound: for every referenced input the two diagonal reference blocks multiply output differences of trace norm one. Thus the half-diamond distance is at most *1/2*.

The inputs $|+\rangle$ and $|-\rangle$ have identical radiation outputs. A decoder of error ε therefore satisfies $1 \le 2\varepsilon$. The identity decoder has error *1/2*, proving the recovery formula.

For the second example, let

$$T = \mathrm{diag}\,(1, e^{i\pi/4}), \qquad V_T|\psi\rangle = T|\psi\rangle_R \otimes |0\rangle_B.$$

The complement is *𝒯_{kb 00}*, and *T^{†}* recovers exactly. The gate is non-Clifford because *TXT^{†} = (X + Y)/√(2)*, which is not a Pauli. For input $|+\rangle$, the output has Bloch components *(1/√(2), 1/√(2), 0)*, outside the convex hull of the six Pauli eigenstates.

The CNOT copies a basis label, not an arbitrary unknown quantum state. There is no violation of no-cloning.

The Choi state of *V_copy* is

$$\frac{|000\rangle + |111\rangle}{\sqrt{2}},$$

a stabilizer state. Thus its recovery obstruction does not require magic in the encoding Choi state.

Even bipartite non-local magic need not cause response

To make ``non-local'' precise for the next counterexample, define a faithful pure-state indicator across a fixed bipartition:

$$D_{\mathrm{NL}}^{A:B}(\psi) = \min_{\text{substack } U_A, U_B}$$

It is zero exactly when the state is locally unitary-equivalent to a pure stabilizer state. We use it only as an indicator: no monotonicity under arbitrary local operations is asserted. Related non-local magic constructions distinguish removable local magic from magic in correlations [Cao2024,CaoEtAl2026].

Lemma. Every reduced density matrix of a pure qubit stabilizer state is proportional to a projector. Its nonzero eigenvalues are equal, and its rank is a power of two.

Proof. For an *n*-qubit pure stabilizer state with stabilizer group *𝒮*,

$$|s\rangle\langle s| = 2^{-n} \sum_{P \in \mathcal{S}} P.$$

Tracing the complementary subsystem leaves only Paulis supported on the retained subsystem *A*:

$$\rho_A = 2^{-n_A} \sum_{P_A \in \mathcal{S}_A} P_A.$$

If *𝒮_A* has *2^r* elements, its group average *2^{−r}∑_{P_A ∈ 𝒮_A} P_A* is a projector. Therefore *ρ_A* has flat nonzero eigenvalues *2^{r − n_A}* and rank *2^{n_A − r}*.

This standard stabilizer structure is also discussed in [FattalEtAl]; the needed statement is proved above.

Theorem. Let *0 < κ < π/4*, and prepare the fixed syndrome

$$|\chi_\kappa\rangle_{R_1B} = \cos\kappa|00\rangle + \sin\kappa|11\rangle.$$

Define

$$V_\kappa|\psi\rangle = |\psi\rangle_{R_0} \otimes |\chi_\kappa\rangle_{R_1B}, \qquad R = R_0R_1.$$

Then

$$\mathcal{C}_\kappa = \mathcal{T}_{\sigma_\kappa}, \qquad \sigma_\kappa = \mathrm{diag}\,(\cos^2\kappa, \sin^2\kappa),$$

so *δ_full = ε_rec^R = 0*.

Nevertheless,

$$D_{\mathrm{NL}}^{R_1:B}(\chi_\kappa) > 0,$$

and the complete encoding Choi state has

$$D_{\mathrm{NL}}^{QR:B}((I_Q \otimes V_\kappa)\Phi_d) > 0.$$

Proof. Tracing *R_0R_1* gives the displayed replacement channel. Discarding *R_1* and returning *R_0* recovers the code exactly.

The two nonzero eigenvalues of the *B*-reduction are unequal. Local unitaries preserve them, whereas a pure stabilizer reduction must have a flat nonzero spectrum by Lemma . Thus no local unitaries convert *χ_{κ}* to a stabilizer state. The local unitary groups are compact and the stabilizer set is finite, so the minimum distance is strictly positive.

The Choi state is *|Φ_d⟩_QR_0 ⊗ |χ_{κ}⟩_R_1B*. For the Choi magic statement, take a qubit code, or more generally *d = 2^k*, with its fixed qubit structure. Across *QR: B*, its *B*-reduction has the same two unequal eigenvalues. The same argument applies.

For the two-qubit syndrome, the indicator can be evaluated:

$$D_{\mathrm{NL}}^{R_1:B}(\chi_\kappa) = \sqrt{1 - \max\left\{\cos^2\kappa, \frac{(\cos\kappa + \sin\kappa)^2}{2}\right\}}.$$

Indeed, the largest overlap with a product state is *cos^2 κ*, whereas the largest overlap with a maximally entangled two-qubit state is *(cos κ + sin κ)^2/2*. These are the two possible Schmidt spectra of a two-qubit pure stabilizer state. Both overlaps are strictly below one for *0 < κ < π/4*.

This counterexample concerns bipartite non-local magic, including in a complete Choi state. It does not identify that quantity with the specialized perturbative tripartite quantity of [CaoEtAl2026]. The fixed syndrome carries no logical-input response.

Corollary. Under the generic prepared-carrier assumptions, neither

$$\text{non-Clifford program} \Rightarrow \delta_{\mathrm{full}} > 0$$

nor

$$\text{positive bipartite non-local magic} \Rightarrow \delta_{\mathrm{full}} > 0$$

is valid. Conversely, *δ_full > 0* does not imply magic.

Proof. Use Proposition and Theorem .

A genuine Clifford criterion and a responding deformation

Pauli compression for a Clifford encoder

Let *C* contain *k* logical qubits and let *A* contain *m* allocated ancilla qubits in *|0^m⟩*. A Clifford unitary *U* gives

$$V|\psi\rangle = U(|\psi\rangle_C|0^m\rangle_A),$$

with a declared output partition *RB*.

Theorem. For each Hermitian residual Pauli *P_B*, write

$$U^\dagger(I_R \otimes P_B)U = s\,P_C \otimes P_A, \qquad s \in \{-1, 1\}.$$

The complete complement is constant if and only if every such Pauli satisfies one of:

• *P_A* has an *X* or *Y* factor, so *⟨0^mrvertP_A |0^m⟩ = 0*;

• *P_C = I_C*.

Proof. Compression gives

$$V^\dagger(I_R \otimes P_B)V = s\,\langle 0^m \mathrm{rvert} P_A\,|0^m\rangle\,P_C.$$

The ancilla expectation is zero when an *X* or *Y* factor is present, and otherwise is one with the phase carried by *s*. Thus every compression is scalar precisely under the stated condition.

Residual Paulis form an operator basis. The criterion for all of them is therefore equivalent to the observable criterion in Theorem .

Cliffordness makes this test algebraic; it does not predetermine its answer. The CNOT copying example fails the test because the residual *Z* compresses to logical *Z*.

A nonresponding Clifford baseline is

$$V_0|\psi\rangle = |\psi\rangle_R|0\rangle_B.$$

A less trivial baseline prepares a Bell syndrome on *R_1B* while keeping *R_0 = C*. Then *B* is fixed in *I_2/2*, even though it is entangled with a radiation syndrome. Constancy does not require a product output or a pure residual.

An exact non-Clifford responding program

On one logical qubit and one allocated residual qubit, define

$$R_y(2\theta) = e^{-i\theta Y},$$

$$U_\theta = \mathrm{kb}\,00 \otimes I + \mathrm{kb}\,11 \otimes R_y(2\theta), \qquad 0 \le \theta \le \frac{\pi}{2}.$$

Initialize *B* in $|0\rangle$ and return the first qubit as *R*. Then

$$V_\theta|0\rangle = |0\rangle_R|0\rangle_B, \qquad V_\theta|1\rangle = |1\rangle_R|b_\theta\rangle_B, \quad |b_\theta\rangle = \cos\theta|0\rangle + \sin\theta|1\rangle.$$

The typed body is

b := new;?(R,B) := ControlledRy_theta(C,b);?returnQ(R,B);

It is a finite semantic program with a specified unitary constant. No exact compilation into a fixed finite gate library is asserted.

Theorem. For ,

$$\mathscr{N}_\theta = \Delta_{\cos\theta}, \qquad \Delta_c(\rho) = \begin{pmatrix} \rho_{00} & c\rho_{01} \\ c\rho_{10} & \rho_{11} \end{pmatrix},$$

and

$$\mathscr{C}_\theta(\rho) = \rho_{00}\mathrm{kb}\,00 + \rho_{11}|b_\theta\rangle\langle b_\theta|.$$

The exact quantities are

$$\mathrm{boxed}\,\delta_{\mathrm{full}}(\theta) = \frac{\sin\theta}{2}, \qquad \varepsilon^R_{\mathrm{rec}}(\theta) = \frac{1-\cos\theta}{2}.$$

For *0 < θ < π/2*, the unitary *U_{θ}* is non-Clifford and its output on the stabilizer input $|+\rangle|0\rangle$ has positive bipartite non-local magic across *R: B*. At *θ = π/2*, the unitary is Clifford but the complement is still nonconstant.

Proof. Partial traces of the two columns in give the displayed channels. The residual pure states have overlap *cos θ*, hence trace distance *sin θ*. Lemma gives the lower bound *δ_full ≥ sin θ/2*.

Let

$$\sigma_\theta = \frac{1}{2}(\mathrm{kb}\,00 + |b_\theta\rangle\langle b_\theta|).$$

Each basis output has distance *sin θ/2* from *σ_{θ}*. For an arbitrary referenced input, the output difference is

$$\sum_{i=0}^{1} X_i \otimes (\sigma_i - \sigma_\theta),$$

where the diagonal reference blocks *X_i* are positive and *∑_i Tr X_i = 1*. The trace norm is therefore at most *sin θ*. This proves the matching half-diamond upper bound.

The radiation channel has

$$\Delta_c = \frac{1+c}{2}\mathrm{id} + \frac{1-c}{2}\mathrm{Ad}_Z.$$

The identity decoder has error *(1 − c)/2*, using an upper triangle bound and the orthogonal Bell outputs of *I* and *Z*. For any decoder of error *ε*, the ideal outputs of $|+\rangle$ and $|-\rangle$ have distance one, while their channel outputs have distance *c*. Contraction and the triangle inequality give *1 ≤ 2ε + c*. This proves the recovery formula.

For input $|+\rangle$, the complete output is

$$|\Psi_\theta\rangle = \frac{|0\rangle|0\rangle + |1\rangle|b_\theta\rangle}{\sqrt{2}}.$$

Its radiation reduction has eigenvalues *(1 + cos θ)/2* and *(1 − cos θ)/2*. For $0 < \theta < \pi/2$, these are nonzero and unequal. Lemma shows that the state is not locally equivalent to a stabilizer state. In particular, *U_{θ}* cannot be Clifford, because it maps a stabilizer input to a nonstabilizer output.

At $\theta = \pi/2$, *R_y(π) = XZ*, and

$$U_{\pi/2} = \mathrm{CNOT}\,\mathrm{CZ},$$

where *CZ* acts first. This is Clifford. On the allocated $|0\rangle$ residual, its code isometry is the copying isometry of Proposition .

This is the properly completed ``responding non-Clifford'' fragment: a specified non-Clifford coupling and a direct residual witness, rather than an inference from gate classification alone. The baseline $\theta = 0$ is nonresponding, and the response is proved on the same code, preparation, and partition.

For small θ,

$$\delta_{\mathrm{full}}(\theta) = \frac{\theta}{2} + O(\theta^3), \qquad \varepsilon^R_{\mathrm{rec}}(\theta) = \frac{\theta^2}{4} + O(\theta^4).$$

The exact relation is

$$\varepsilon^R_{\mathrm{rec}} = \frac{1 - \sqrt{1 - 4\delta^2_{\mathrm{full}}}}{2}.$$

This relation belongs to this family, not to arbitrary encodings.

Magic on $|\Psi_{\theta}\rangle$ vanishes at both endpoints, while leakage is zero only at the first endpoint. Thus even in this entangling family, leakage is not a faithful magic measure.

A general sufficient response condition

Theorem. Let *V_0: C → RB* have constant complement, let *K = K^{†}* be bounded, and define

$$V_\lambda = e^{-\mathrm{i}\lambda K} V_0.$$

Fix code states *ρ_0*, *ρ_1*, put *X = ρ_0 − ρ_1*, and choose a bounded Hermitian residual observable *O* with $\|O\| \le 1$. Suppose

$$m = \mathrm{Tr}\left[O\left(-\mathrm{i}\mathrm{Tr}_R\left[K, V_0 X V_0^\dagger\right]\right)\right] \neq 0.$$

Then

$$\delta_{\mathrm{full}}(V_\lambda) \ge \max\left\{0, \frac{|m|\,|\lambda|}{4} - \|K\|^2\lambda^2\right\}.$$

In particular, the complement is nonconstant for

$$0 < |\lambda| < \frac{|m|}{4\|K\|^2}.$$

Proof. The derivative of the conjugated trace-class operator is *−i[K, V_0XV_0^{†}]*. The double commutator has trace norm at most $4\|K\|^2\|X\|_1$. Taylor's integral remainder is therefore bounded by $2\lambda^2\|K\|^2\|X\|_1 \le 4\lambda^2\|K\|^2$.

The baseline residual image of *X* is zero by constancy. Hence

$$|\mathrm{Tr}\,(O\mathscr{C}_\lambda(X))| \ge |m|\,|\lambda| - 4\lambda^2\|K\|^2.$$

Apply . The assumption $m \neq 0$ entails $\|K\| > 0$.

The relevant additional premise is a nonzero residual matrix element of the coupling. A label such as ``interacting,'' ``non-Gaussian,'' or ``non-Clifford'' is not that premise. A vanishing first-order witness also does not rule out higher-order response.

Readout, forgetting, and the complete apparatus

The external residual program

For a finite encoded residual, the observer may explicitly add

$$P_{\text{forgetB}} : Q_R \otimes Q_B \to Q_R$$

with body

x := Enter(B);?dropBit(x);?returnQ(R);

This is an external operation, not a conclusion about autonomous evolution.

Theorem. For every joint state, including an external reference,

$$\sum_x (\langle x|_B \otimes I)\rho(|x\rangle_B \otimes I) = \mathrm{Tr}_B \rho.$$

Consequently,

$$P_{\text{forgetB}} \mathscr{V} = \mathscr{N},$$

with exactly the pre-existing radiation recovery error.

More generally, a trace-preserving local operation on B, with all its outputs ignored, leaves the untouched reference and R unchanged.

Proof. Partial trace is the sum of the diagonal blocks in any orthonormal basis. For a general local channel, its Heisenberg adjoint is unital. It therefore leaves every observable $A \otimes I_B$ on the untouched factors unchanged.

Thus the phrase ``empty the bin'' is only operational forgetting at this interface. It does not prove annihilation of the physical residual or its apparatus.

For an infinite residual, partial trace remains well defined. The theorem does not provide a finite program that measures infinitely many modes.

Proposition. For the instrument , define

$$W_{\text{read}}|\psi\rangle = \sum_x |x\rangle_K |x\rangle_E M_x |\psi\rangle.$$

This is an isometry. Tracing E gives the observable destructive readout instrument. The complete output preserves the input globally.

Proof. Completeness of the M_x gives $W_{read}^{\dagger}W_{read} = I$. Tracing the orthogonal E-labels removes cross-outcome terms. Apply Theorem .

For a fully measured qubit,

$$a|0\rangle + b|1\rangle \longmapsto a|0\rangle_K|0\rangle_E + b|1\rangle_K|1\rangle_E.$$

The phase is absent from each classical marginal but retained jointly. The carrier K in this coherent dilation is a quantum system; its observable *Bit* interface arises after E is omitted.

This complete apparatus ledger is the distinction emphasized in [CheungHawk04B,CheungEm03C]. Operational discard may move a record carrier to an inaccessible environment in a unitary realization; it does not remove that environment from the complete preservation claim.

Theorem. If the only retained output of a complete process is classical, every subsequent deterministic reconstruction of C has error at least

$$1 - d^{-1}.$$

The same bound holds for every measure-and-prepare channel followed by a decoder. Computational readout followed by basis-state preparation attains it.

Proof. A finite classical-output channel has the form

$$X \longmapsto \sum_x \mathrm{Tr}\,(F_x X)|x\rangle\langle x|.$$

A decoder prepares one state for each outcome, giving a measure-and-prepare channel. Its Bell-input output is

separable.

For a normalized product vector $u \otimes v$,

$$|\langle \Phi_d, u \otimes v \rangle|^2 \le 1/d.$$

Convexity gives the same Bell-overlap upper bound for separable states. Testing the Bell projector gives error at least $1 - 1/d$.

Computational dephasing is

$$\Delta_d = \frac{1}{d} \sum_{k=0}^{d-1} \mathrm{Ad}_{Z^k}.$$

The d Bell outputs are orthogonal, yielding distance $1 - 1/d$; the triangle inequality gives the matching upper bound.

The entanglement-breaking characterization is established in [HorodeckiShorRuskai]. The quantitative statement is proved locally. A surviving quantum memory or supplied quantum side information changes the hypothesis.

Residual measurement outcomes supplied to the observer may assist quantum recovery. For example,

$$V|\psi\rangle = \frac{|\psi\rangle_R|0\rangle_B + Z|\psi\rangle_R|1\rangle_B}{\sqrt{2}}$$

has dephasing radiation. Measuring B gives a uniform bit x; applying Z^x to R recovers exactly. Dropping the bit without using it leaves dephasing. The bit alone contains no diary. This is an access change, not a record-only quantum decoder.

The four-Majorana control is not a magic certificate

The independently specified model

Take four Majoranas with

$$\{\chi_i, \chi_j\} = 2\delta_{ij} I, \qquad S_j = \mathrm{i}\chi_{2j-1}\chi_{2j},$$

and freeze

$$H_S = J(I - S_1 S_2), \qquad A = \frac{S_1 + S_2}{2}, \qquad J > 0.$$

For $0 < \mu < \Lambda < \infty$, let

$$\mathfrak{h} = L^2([\mu, \Lambda], \mathrm{d}\omega), \qquad (h_B f)(\omega) = \omega f(\omega),$$

$$H_B = \mathrm{d}\Gamma(h_B), \qquad f(\omega) = \frac{1}{\sqrt{2\omega}}, \qquad H = H_S + H_B + gA \otimes \Phi(f).$$

The Hamiltonian is specified independently of Enter.

The field estimates

$$\|a(f)\psi\| \le \left\| h_B^{-1/2} f \right\| \left\| H_B^{1/2} \psi \right\|,$$

$$\left\| a^\dagger(f)\psi \right\|^2 = \|a(f)\psi\|^2 + \|f\|^2 \|\psi\|^2$$

give arbitrarily small relative bound of the interaction with respect to H_B. Since H_S and A are bounded, Kato–Rellich gives self-adjointness on $D(H_B)$ [Kato]. The finite cutoffs make the displayed norms finite.

Prepare the source in $|11\rangle_S$ and the bath in vacuum. Swap an even-parity two-mode diary into the source:

$$|0\rangle_C \mapsto |00\rangle_S, \qquad |1\rangle_C \mapsto |11\rangle_S.$$

The displaced carrier is the fixed state $|11\rangle_D$. The logical source states s_0, s_1 have zero source energy and coupling eigenvalues $a_0 = -1$, $a_1 = 1$.

For a declared packet projection $P_{\mathfrak{r}}$, set

$$\eta_i(t) = g a_i \frac{e^{-i\omega t} - 1}{\omega} f(\omega),$$

$$\eta_{i,R} = P_\tau \eta_i, \qquad \eta_{i,U} = (I - P_\tau)\eta_i,$$

$$c_R = \exp\left[-\frac{1}{2}\|\eta_{0,R} - \eta_{1,R}\|^2\right], \qquad z_R = \sqrt{1 - c_R^2}.$$

Radiation is the collected bath; the source, carrier, and uncollected bath form the complete residual.

Theorem. The complete code vectors, up to a common phase, are

$$V_t|i\rangle = |\eta_{i,R}\rangle_R \otimes |s_i\rangle_S |11\rangle_D |\eta_{i,U}\rangle.$$

They give

$$\mathcal{N}_t(\rho) = \rho_{00}|\eta_{0,R}\rangle\langle\eta_{0,R}| + \rho_{11}|\eta_{1,R}\rangle\langle\eta_{1,R}|,$$

and the exact formulas

$$\text{boxed}\, \varepsilon_{\text{rec}}^R(t) = \frac{3 - z_R}{4}, \qquad \varepsilon_{\text{rec}}^B(t) = \frac{1 - c_R}{2}, \qquad \delta_{\text{full}}(t) = \max\left\{\frac{1}{2}, \frac{1 + 2c_R}{4}\right\}.$$

In particular, *ε_rec^R ≥ 1/2* and *δ_full ≥ 1/2*. The complete output is exactly reversible.

Proof. On source eigenvalue *a_i*, let *q_i = g a_i h_B^{−1} f*. The Weyl identity gives

$$H_B + g a_i \Phi(f) = \mathcal{W}(q_i)^\dagger H_B \mathcal{W}(q_i) - g^2 a_i^2 \left\| h_B^{-1/2} f \right\|^2 I,$$

where *𝒲(q) = e^{a^{†}(q) − a(q)}*. The vacuum therefore evolves to the coherent amplitude *(e^{−ith_B} − I)q_i = η_i(t)*. The phase is common for *a_i = ±1*. Orthogonal packet decomposition factors the coherent state. Tracing the orthogonal source labels gives the radiation channel.

Its two pure output states have trace distance *z_R*. Their optimal equal-prior discrimination probability is *(1 + z_R)/2*, obtained by the positive part of their difference. For any radiation decoder, its Bell overlap is

$$F_e = \frac{1}{4}[\langle 0|\mathcal{D}(\sigma_0)|0\rangle + \langle 1|\mathcal{D}(\sigma_1)|1\rangle] \le \frac{1 + z_R}{4}.$$

The two diagonal decoder effects constitute a discrimination POVM. The Bell-projector test gives error at least *(3 − z_R)/4*.

Bath parity interchanges the two states, so an optimal discrimination POVM can be symmetrized to have equal conditional errors. Measurement followed by preparation of the inferred basis state gives

$$z_R \Delta_Z + (1 - z_R)\mathcal{T}_{I_2/2}.$$

Its Pauli probabilities are

$$p_I = p_Z = \frac{1 + z_R}{4}, \qquad p_X = p_Y = \frac{1 - z_R}{4}.$$

Orthogonal Pauli Bell outputs and the triangle inequality give half-diamond error *1 − p_I*, proving the radiation formula.

Define the isometry

$$L_t|i\rangle = |s_i\rangle_S |11\rangle_D |\eta_{i,U}\rangle.$$

Then

$$\mathcal{C}_t = L_t \Delta_{c_R}(\,\cdot\,) L_t^\dagger.$$

Retracting the range of *L_t* gives the dephasing recovery error *(1 − c_R)/2*, by the same upper bound and $|+\rangle$, $|-\rangle$ lower bound used in Theorem .

Isometric embedding and CPTP retraction preserve leakage. For *Δ_c*, Pauli covariance and convexity show that twirling any candidate replacement state to *I_2/2* cannot increase its distance. Relative to that replacement, the Pauli coefficients, in order *I, Z, X, Y*, are

$$\frac{1+2c}{4},\quad \frac{1-2c}{4},\quad -\frac{1}{4},\quad -\frac{1}{4}.$$

The half-diamond norm is half the sum of their absolute values: a Bell input gives the lower bound and the triangle inequality gives the upper bound. Hence

$$\delta_{\text{full}} = \frac{3+2c+|1-2c|}{8} = \max\left\{\frac{1}{2}, \frac{1+2c}{4}\right\}.$$

The complete columns are orthonormal, so global recovery follows from Theorem .

The radiation obstruction is [CheungHawk04A]; the residual formulas are the refinements in [CheungHawk04B]. They are rederived here.

The baseline prevents a magic-origin inference

At $g = 0$, or at $t = 0$, collected radiation is vacuum and $c_R = 1$. Therefore

$$\varepsilon^{R}_{\text{rec}} = \frac{3}{4}, \qquad \varepsilon^{B}_{\text{rec}} = 0, \qquad \delta_{\text{full}} = \frac{3}{4}.$$

Residual non-constancy is already maximal for a qubit at the initial interface: the diary is retained in the inaccessible source. Nonzero coupling is not the cause of that pre-existing non-constancy.

As collected coherent states become more distinguishable, the radiation error decreases and residual leakage can decrease. Thus this control cannot establish that added non-Clifford resources force added leakage.

No fixed qubit Pauli dictionary for the entire bosonic dynamics has been declared, and no magic measure has been evaluated for it. The control is commuting and uses no scrambling or out-of-time-order estimate. It is a recovery obstruction, not a certificate of non-local magic or geometric response.

If all bath modes are accessible,

$$c_R(t) = \exp\left[-2g^2\int_{\mu}^{\Lambda}\frac{1-\cos(\omega t)}{\omega^3}\mathrm{d}\omega\right].$$

For a restricted packet space, a nonzero coupling need not produce nonzero collected displacement at every chosen time.

The source energy remains zero, while

$$E_B(t) = g^2\int_{\mu}^{\Lambda}\frac{1-\cos(\omega t)}{\omega^2}\mathrm{d}\omega, \qquad E_I(t) = -E_B(t).$$

This follows from coherent bath energy and total-energy conservation. It is dressing of an initially undressed preparation, not a gravitational evaporation law.

Finite certificates and what they actually establish

Choi and complementary-channel comparisons

Lemma. For channels of input dimension d,

$$d_{\diamond}(\Phi,\Psi) \le \min\left\{1, \frac{d}{2}\|J_{\mathrm{n}}(\Phi) - J_{\mathrm{n}}(\Psi)\|_1\right\}.$$

Proof. A normalized pure referenced input has the form $(L \otimes I)|\Phi_d\rangle$, with $Tr\ (L\^\{\dagger\}L) = d$ and $\|L\|\^2 \le d$. Multiplying the normalized Choi difference on both sides by L bounds its trace norm by d times the Choi trace norm. Convexity handles mixed inputs.

For finite complementary channels, the KSW information–disturbance theorem [KSW] implies, in the present half-diamond convention,

$$\frac{\delta^2_{\text{full}}}{2} \le \varepsilon^{R}_{\text{rec}} \le \sqrt{2\delta_{\text{full}}}.$$

Indeed, in full-diamond normalization the recovery-to-leakage direction gives

$$\inf_{\sigma}\|\mathscr{C} - \mathscr{T}_{\sigma}\|_{\diamond} \le 2\sqrt{\inf_{\mathscr{D}}\|\mathscr{D}\mathscr{N} - \mathrm{id}\|_{\diamond}},$$

and the opposite direction gives the analogous bound with recovery and leakage exchanged. Division by two yields . This is a channel conversion, not a magic theorem or a dynamical estimate.

Theorem. Suppose a finite prepared proxy is embedded access-preservingly in a common output space and satisfies

$$d_{\diamond}(\mathscr{V}, \widehat{\mathscr{V}}) \le b.$$

Then

$$\left|\delta_{\mathrm{full}} - \widehat{\delta}_{\mathrm{full}}\right| \le b, \qquad \left|\varepsilon^{R}_{\mathrm{rec}} - \widehat{\varepsilon}^{R}_{\mathrm{rec}}\right| \le b.$$

If a validated proxy pair witness gives

$$\widehat{x} = d_{\mathrm{tr}}(\widehat{\mathscr{C}}(\rho_0), \widehat{\mathscr{C}}(\rho_1)),$$

then

$$\delta_{\mathrm{full}} \ge \max\{0, \widehat{x}/2 - b\}.$$

For a specified proxy replacement state, put

$$e = \frac{1}{2}\left\| J_{\mathrm{n}}(\widehat{\mathscr{C}}) - \frac{I_Q}{d} \otimes \widehat{\sigma} \right\|_1.$$

Then

$$\delta_{\mathrm{full}} \le \min\{a_d, de + b\},$$

and a finite proxy decoder has a full extension satisfying

$$d_{\diamond}(\mathscr{D}\mathscr{N}, \mathrm{id}_C) \le \min\{a_d, \sqrt{2de} + b\}.$$

The last bound means that one may choose the better of the extended proxy decoder and the constant decoder.

Proof. Partial trace gives allowance b for both reduced channels. A local isometric embedding has a CPTP retraction

$$Y \mapsto J^{\dagger} Y J + \mathrm{Tr}\,((I - JJ^{\dagger})Y)\tau.$$

Embedding and retracting decoders and replacement states preserve the tasks. Lemma therefore gives stability.

The pair-witness lower bound follows from Lemma and leakage stability. The Choi estimate gives proxy leakage at most de. Equation supplies a finite decoder of error at most $\sqrt{(2de)}$. Extend it off the proxy support by fixed-state preparation. Contraction adds b. Lemma supplies the cap.

A specified decoder can instead be certified by its normalized logical Choi error r, giving allowance $dr + b$. CPTP feasibility must be validated, including positivity and trace preservation on the entire finite input space. An optimization objective alone is not such a certificate.

Finite-channel certificates concern response and recovery. They do not certify magic unless a separate fixed-frame resource calculation is supplied.

A finite-time microscopic allowance

For completeness, the regulator used by the source–scalar architecture can be stated independently of the magic claims. Let the source be finite dimensional, let H_S be bounded, and let

$$H = H_S + \mathrm{d}\Gamma(h_B) + \sum_a g_a A_a \otimes \Phi(f_a),$$

with self-adjoint bounded A_a, vacuum bath preparation, and $\mu I \le h_B \le \Lambda I$. Define

$$G = \sum_a |g_a| \|A_a\| \|f_a\|, \qquad Z_m(x) = \sum_{\ell = m+1}^{\infty} \frac{x^{\ell}}{\sqrt{\ell!}}.$$

Proposition. For finitely many declared packets, occupation cutoff m, frequency bins of width at most Δ, and $0 \le t \le T$, one can construct a finite active proxy with

$$d_\diamond(\mathscr{V}_t, \widehat{\mathscr{V}}_t) \le \min\left\{1, 2Z_m(2GT) + \frac{Tm\Delta}{2}\right\}.$$

The embedding preserves the accessible–inaccessible partition.

Proof. In interaction picture, on occupation at most k,

$$\|V_I(s)\psi\| \le 2G\sqrt{k+1}\,\|\psi\|.$$

An ℓ-fold product starting from vacuum has norm at most $(2G)^\ell\sqrt{(\ell!)}$. Ordered integration gives Dyson-term bound $(2Gt)^\ell/\sqrt{(\ell!)}$, uniformly over code–reference inputs. These series converge also after multiplication by any fixed occupation power; since h_B is bounded, termwise differentiation on prepared vectors identifies the series with the self-adjoint evolution.

Compression to occupation at most m leaves all terms through order m unchanged. Higher terms in either series obey the same bound, giving prepared-isometry difference at most $2Z_m(2GT)$.

Replace frequency by the midpoint of each bin. The one-particle operator difference has norm at most $\Delta/2$; on occupation at most m, its second quantization has norm at most $m\Delta/2$. Duhamel's formula adds $Tm\Delta/2$.

The span of the bin-restricted form factors and packet functions is finite dimensional and invariant under the midpoint operator. Modes orthogonal to it stay in vacuum. Its total-occupation cutoff is therefore a finite active proxy. The total cutoff embeds in separate collected and uncollected cutoff Fock spaces; it is not equal to their full tensor product. The source and displaced carrier remain in the residual.

Lemma converts prepared-isometry error to the complete-channel allowance.

This is the finite-time comparison discipline of [CheungHawk02E,CheungHawk04B]. It does not remove the infrared or ultraviolet cutoffs. Separate finite qubit dictionaries preserve the channel task but do not construct a local physical receiver or transport a magic classification automatically.

Compatible half-diamond preparation, collection, and implementation allowances telescope under CPTP composition [CheungEn02B,CheungEn02C]. The flagged-cutoff equalities in [CheungEn02E] retain their commuting-state hypotheses. A discarded probability is not a generic coherent-channel error.

What would be needed for a geometric interpretation

A declared geometric subsystem is additional data

Suppose a physical model identifies

$$B = G \otimes E$$

and specifies a collection $\mathcal{O}_G$ of bounded Hermitian observables intended to represent geometric quantities. Define

$$\mathscr{C}_G = \mathrm{Tr}_E\,\mathscr{C}.$$

This declaration is stronger than merely naming B ``geometry."

Proposition. If some $O_G \in \mathcal{O}_G$, with $\|O\|_G \le 1$, and some code states satisfy

$$\mathrm{Tr}\,(O_G \mathscr{C}_G(\rho_0 - \rho_1)) \neq 0,$$

then the complete residual is nonconstant and exact whole-code recovery from R is impossible.

The converse is false.

Proof. Lift the observable to $O_G \otimes I_E$ and apply Lemma and Theorem .

For the converse, take

$$V|\psi\rangle = |0\rangle_R|\mathrm{vac}\rangle_G|\psi\rangle_E.$$

The complete residual recovers the diary and is nonconstant, but $\mathscr{C}_G = \mathscr{T}\{|vac\rangle\langle vac|\}$. Every declared geometric observable is input independent.

This counterexample also shows why vanishing source energy next to an inaccessible diary memory does not imply residual independence [CheungHawk02E]. Choose $H_G|vac\rangle = 0$; its energy is always zero despite

complete residual response.

Entropic rigidity is not a universal geometry theorem

In finite dimension, the exact normal form gives

$$S(\mathcal{N}(\rho)) = S(\rho) + S(\sigma_K).$$

The additive syndrome entropy is independent of the logical input. This follows directly from tensor-product entropy and isometric invariance.

Conversely, a selected entropy does not locate the code. For

$$V_{\text{good}}\psi = \psi_R|0\rangle_B, \qquad V_{\text{bad}}\psi = |0\rangle_R\psi_B,$$

both radiation entropies vanish on every pure code input. Their radiation recovery errors are *0* and *a_d*. An entropy identity alone is not the complete complementary-channel criterion.

Cao's area-operator result concerns stabilizer codes with the specified complementary operator-algebra structure, not arbitrary Clifford channels [Cao2024]. The skewed-code analysis of Cao and collaborators relates leading proto-area entropy corrections to a specially defined perturbative tripartite Choi magic under its subsystem and perturbation assumptions [CaoEtAl2026]. Neither statement equates generic complete residual non-constancy with geometric backreaction.

The syndrome counterexample in Theorem has bipartite magic but no logical response. It therefore does not contradict a theorem whose resource is a more restricted matter–geometry tripartite coupling. It demonstrates why those restrictions matter.

The corrected conditional bridge

Corollary. Fix a physical code, complete preparation, unitary deformation, and access partition. Suppose:

• a residual subsystem or observable is justified as geometric by an independent same-model dictionary;

• the deformation has a nonzero response witness in that observable, such as the coefficient in Theorem ;

• any claimed magic resource is independently defined and certified in the same carrier convention. Then the certified observable responds to the declared code inputs, the complete complement is nonconstant, and exact whole-code radiation recovery is obstructed.

The magic certificate does not replace the response witness or the geometric dictionary.

Proof. The response witness and the dictionary identify the intended observable variation. Proposition gives nonconstancy and the exact-recovery obstruction.

A metric equation, stress–energy identification, backreaction law, and physical endpoint require further same-model constructions. None is supplied by this corollary.

Likewise, a continuum claim about vacuum magic is not a finite carrier leakage theorem. For example, the thermodynamic-limit magic analysis of [Moosa2026] concerns a different algebraic setting. Even if a particular vacuum resource theorem is applicable, a bridge to this task still needs a preparation channel, a finite dictionary, a complete residual, and a response estimate. No intrinsic-magic claim for a Lorentz-invariant vacuum is used as a premise here.

Endpoint continuity and recurrence

Access-preserving limits

For time-dependent partitions *R_tB_t*, let

$$J_{R,t} : R_t \to R_\infty, \qquad J_{B,t} : B_t \to B_\infty$$

be local isometries, and put

$$\overline{V}_t = (J_{R,t} \otimes J_{B,t})V_t.$$

These do not move an inaccessible diary factor into radiation.

Theorem. If $\forall_t|i\rangle \to V_{\{\infty\}}|i\rangle$ on a code basis, then *V_{∞}* is an isometry and

$$a_t = \|\overline{V}_t - V_\infty\| \to 0.$$

Complete and reduced channels converge in half-diamond distance with allowance *a_t*, and

$$|\delta_{\text{full}}(t) - \delta_{\text{full}}(\infty)| \le a_t.$$

Thus a proved limit *δ_full(t) → 0* gives exact endpoint recovery from radiation.

Proof. Finite-dimensionality of the code gives

$$\|\overline{V}_t - V_\infty\|^2 \le \sum_i \|(\overline{V}_t - V_\infty)|i\rangle\|^2 \to 0.$$

Inner products pass to the limit. Lemma gives channel convergence. Local embeddings preserve leakage by the CPTP retraction used in Theorem . Zero endpoint leakage implies exact constancy by Lemma ; apply Theorem .

This is the access-preserving endpoint interface in [CheungHawk04A,CheungHawk04B]. A unitary swapping *R* and *B* changes the recovery task and is not an admissible local identification.

If the endpoint residual is fixed pure, endpoint radiation is an isometric encoding. A mixed fixed residual gives the syndrome normal form. An infinite endpoint syndrome need not admit a finite decoder program.

Fixed finite proxies return

Proposition. For a fixed finite-dimensional autonomous Hamiltonian, there are arbitrarily late times at which its prepared complete channel is arbitrarily close to its initial channel.

If initially accessible radiation is fixed vacuum and the complete prepared map is an isometry, then

$$\limsup_{t \to \infty} \varepsilon^R_{\text{rec}}(t) = a_d, \qquad \limsup_{t \to \infty} \delta_{\text{full}}(t) = a_d.$$

These statements do not assert positive leakage at every time.

Proof. The finitely many spectral phases evolve in a compact torus. Simultaneous recurrence gives arbitrarily late unitary returns to the identity; periodic cases use multiples of a period. Lemma gives prepared-channel returns.

At time zero, radiation is constant, so its recovery error is *a_d*. The complete isometry with pure fixed radiation has an isometric residual encoding, whose leakage is *a_d* by Lemma and isometric invariance. Distance stability gives both values along the recurrence sequence. The universal upper bounds in Lemma give the limsup equalities.

This is the fixed-proxy obstruction of [CheungHawk02E], refined using the complete-channel distances. It gives recurrent nonresponse failure when the initial task has that failure. It neither proves monotone leakage nor identifies its origin as magic.

A finite acyclic program denoting *U(t)* terminates at each fixed *t*. That syntactic fact proves no asymptotic endpoint, no permanent release, and no disappearance of *B*. A finite-time regulator must precede a controlled outgoing limit.

Completed program and destination ledger

toprule Declared program or condition Proved channel statement What is not inferred

midrule endfirsthead toprule Declared program or condition Proved channel statement What is not inferred

midrule endhead bottomrule endfoot

Identity or tested Clifford baseline Complete complement constant; quantum radiation return is exact. No geometric model follows from Cliffordness.

Clifford CNOT copying body *δ_full = ε_rec^R = 1/2*; logical basis information enters *B*. Magic is not necessary for residual response.

Local non-Clifford logical body Complement constant; inverse logical unitary recovers exactly. A non-Clifford gate does not force leakage.

Fixed non-local magical syndrome Complement fixed mixed; logical register remains exactly recoverable. Bipartite non-local magic is not sufficient for response.

Controlled-*R_y* body *δ_full = sin θ/2*; *ε_rec^R = (1 − cos θ)/2*. The formulas are model-specific, not universal

magic laws.

Four-Majorana control $\varepsilon_{rec}^{R} = (3 - z_R)/4 \geq 1/2$; complete output reversible. No scrambling, magic, or gravitational certificate.

Local quantum decoder Returns the code register; work is explicitly consumed. Enter on the diary is not the decoder.

Residual Enter with record supplied Access becomes *RK*; outcome-assisted correction may help. Bits alone do not replace an unknown diary.

Residual Enter then dropBit Observable channel is exactly $\mathcal{N}$. No permanent physical deletion or additional radiation erasure.

Constant radiation Complete residual alone exactly recovers the code. Global preservation alone does not imply this localization.

Nonconstant complete residual Some logical information is distinguished by *B*. Residual-only recovery of the whole code need not hold.

Constant endpoint complete residual Exact endpoint radiation recovery, with access-preserving outputs. No gravitational endpoint without a physical construction.

toprule Plan step Completed or corrected deliverable

midrule 1 Finite prepared-carrier interface and reference-preserving destructive Born instrument. No field preparation is derived from typing.

2 Global restore and exact constant-complement criterion. A Clifford baseline requires the Pauli test, not its label.

3 The unconditional magic implication is disproved. A non-Clifford responding family and a general response witness are constructed instead.

4 Joint preservation, residual-only localization, observation, and operational forgetting are separated. No deletion theorem is claimed.

5 Access-preserving endpoint continuity and arbitrarily late finite-proxy returns are proved without promoting termination to evaporation.

6 OpenAI and XIV/XV/XVI comparisons remain documentary separations; they supply no recovery, magic, geometry, or admission premise.

bottomrule

Conclusion

The generic implication from non-Clifford or bipartite non-local magic to a nonconstant complete residual is false. The converse implication is false as well. The exact invariant governing whole-code radiation recovery is constancy of the complete complementary channel.

A specified non-Clifford deformation can produce residual response: the controlled-rotation construction proves this with exact distances, and the first-order observable theorem supplies a general sufficient condition. The coupling witness, not the resource label alone, establishes the conclusion.

Residual response is not geometric backreaction. A geometric interpretation needs a justified subsystem or observable dictionary, a matter-input identification, and same-model physical dynamics. The supplied plan cannot replace these hypotheses by terminology.

Enter remains destructive readout after a finite carrier exists. It is not unitary evolution, quantum decoding, or physical deletion. Complete unitary apparatus outputs retain the information omitted from the observer's classical interface.

The completed mathematical release consists of the local channel theorems, the Clifford test, the magic counterexamples, the responding family, the quantitative witness, the control accounting, and the endpoint and recurrence distinctions. No physical field-to-memory receiver, metric response law, gravitational endpoint, or admitted analytic quantum coefficient is established.

Declarations

Verification. The source supplies analytical arguments. No successful pdflatex execution, numerical optimization, proof-assistant verification, independent technical review, or journal acceptance is asserted.

Authorship and source accessibility. Author metadata follows the supplied Cheung predecessor sources. The responsible author should confirm authorship and archival accessibility of the unpublished comparisons before submission. Missing publication metadata and persistent identifiers are not invented.

Originality and scope. Exact correction, no-hiding, Stinespring dilation, stabilizer structure, and information–disturbance are established theories. The present manuscript does not claim priority for them. Its explicit comparisons do not identify different models, resource conventions, or geometric observables.

appendix

Documentary analytic boundaries

The supplied analytic excerpts concern a different distributional characteristic-composition problem. Their kernels, carrier, signs, cutoffs, and observable are not identified with C, R, B, V, or U_{θ}.

The supplied XIV-H release stops at its H2 transition-amplitude requirement. XIV-I constructs its named internal amplitude module at I2 but does not construct the complete joint characteristic map at I3 [CheungXIVH,CheungXIVI]. XIV-J supplies reconstruction descent and an off-characteristic reference for $K_6 + K_8 + K_9$, not the full six-branch map on tests meeting the characteristic locus [CheungXIVJ].

The necessary algebraic descent condition can be proved independently.

Lemma. Let $Rec: P \to K$ be a surjective linear map. A linear candidate $M: P \to D$ factors uniquely through K if and only if

$$\ker \mathrm{Rec} \subseteq \ker \widetilde{M}.$$

Proof. Necessity follows from factorization. Conversely, define $M(Rec\ p) = M(p)$. Presentations of the same reconstruction differ by a null presentation, so this is well defined and unique.

This interface is the one recorded in [CheungXIVJ]. It does not construct a characteristic product. The identity $q\delta^{\{'\}}(q) = -\delta(q)$ likewise shows that presentation degree need not descend to an intrinsic symbol. It does not invalidate an already proved internal reconstruction.

The remaining documentary chain is

nt characteristic map
on and controlled remainder
m transverse estimates
ntact compatibility
output wavefront theorem
of the required coefficient.

Noarrowissuppliedbyfinitetyping, Enter, aresourcelabel, oraresidualwitness. Underthesuppliedcontract, theadmission − dependentexitwouldrequiretheunassertedmembership

These symbols identify that separate documentary problem; they are not objects constructed here. XV-B contains abstract lifting and extension interfaces but does not admit the inherited coefficient [CheungXVB]. Thus its admission-dependent exit is not opened by En04-A.

The supplied XVI-N obstruction is conditional on its specified massless instance and route hypotheses [CheungXVIN]. It neither constructs the missing characteristic map nor discharges a universal analytic completion. The corresponding broader obligations are not discharged here.

The analytic continuation is not the parallel relative-classical XIV-A–XIV-B branch. XIII-E's alternative route is not imported. Later continuation or patch labels do not constitute completed maps. These statements report the supplied scope, not universal nonexistence theorems.

In the supplied terminology, O.1 remains NOT_CLOSED. This is a documentary status label, not a new mathematical object closed by the present channel proofs.

OpenAI catalogue: separation, not premise

The supplied companion record attributes the following family descriptions to OpenAI [SuppliedCatalogue]. Their

proofs, attribution, and publication status are not independently verified here. No family is a premise or an evaluation of U_{θ}, the four-Majorana control, or a gravitational model.

toprule Family Supplied topic Refused transfer

midrule 227 Sherrington–Kirkpatrick mixing, dynamical transitions, and critical autocorrelations No quantum scrambling, OTOC evaluation, or four-Majorana recovery certificate.

279 Exact quantum factoring over a fixed finite gate set No compilation or complexity theorem for the admitted unitary constants.

274 Moore's parity conjecture for QAC^0 No limitation on this differently specified typed fragment and no residual-response evaluation.

276 Classical capacity of qubit generalized amplitude-damping channels No complete-complement decoder and no substitute for exact whole-code correction.

bottomrule

In particular, ``Family 227 by OpenAI" is not an estimate used in the exact control proof, and ``Family 279 by OpenAI" does not justify exact synthesis of a controlled continuous-angle rotation. OpenAI catalogue entries remain documentary separations. O.1 stays NOT_CLOSED; no quantum coefficient is admitted; XV-B's admission exit is not opened; and the separate XVI-N obligations are not discharged.

A submission package for a future physical bridge

A positive same-model magic-to-geometric-response claim should archive at least:

• the physical code and complete preparation or purification;

• the field-to-carrier interface and approximation topology;

• the frozen Pauli or other resource convention;

• the complete coherent evolution and access partition;

• the precise magic quantity, free operations, and validated resource certificate;

• the full logical matrix-unit channels, not only populations;

• a residual response witness and its error allowance;

• independently justified geometric observables and matter labels;

• the same-model dynamical or constitutive comparison connecting those observables to geometry;

• a complete quantum decoder or a proved recovery obstruction;

• separate quantum-return and classical-observation programs;

• all syndrome, failure, work, measurement, and discard outputs;

• compatible regulator and implementation allowances;

• a justified common time interval or access-preserving endpoint.

A vacuum magic claim, a non-Clifford gate count, a selected entropy, a classical record, or a finite-program termination theorem does not replace this package.

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OpenAI, as attributed in the supplied companion record, *Enter Calculus Companion: OpenAI family catalogue excerpts*. Supplied documentary record describing Families 227, 279, 274, and 276; the supplied Family 279 record is labelled rem:oai-279 and dated 7 October 2026. Attribution, proofs, and publication status are not independently verified here. No family is a theorem premise.