En07-C Hawking Radiation under Destructive Enter

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Version 1

Abstract

Hawking's 1975 calculation identifies late-time particle emission for quantum fields on a classical collapse background, with a thermal factor modified by absorption probabilities. It does not construct a microscopic quantum-gravitational evaporation channel through the final Planck-scale regime. This paper supplies a self-contained finite-interface comparison under the Philosophy of Enter. A declared code is embedded isometrically into accessible and complete residual outputs. Exact reference-preserving recovery from the accessible output is equivalent to constancy of the complete complement, with an explicit deterministic decoder and a residual-rank dimension bound. Destructive basis readout followed by forgetting equals partial trace, whereas its complete coherent dilation remains isometric. Nearly constant accessible channels give recovery lower bounds; nearly constant complete complements give recovery upper bounds, with the approximate statement explicitly importing the Kretschmann–Schlingemann–Werner theorem. Finite counterexamples separate thermal or spectral data, selected entropy, conserved energy, and termination from quantum return. An access-preserving endpoint theorem combines convergence, complement, and receiver allowances under four same-model obligations. Neither those gravitational obligations nor an outgoing gravitational receiver is constructed here. Companion manuscripts are documentary comparisons, and no inherited gravitational quantum coefficient is admitted.

Bibliographic record

Title	En07-C Hawking Radiation under Destructive Enter
Author	Cheung, N.
Publication date	2026
Institution	Enter
Report number	07-En07-C
Publisher	Cheung Ngo Medical Limited
First page	1
Last page	21
Language	en

Keywords: Hawking radiation; quantum error correction; complementary channel; diamond distance; destructive measurement; Landauer principle; conditional evaporation endpoint.

Scope and corrections to the proposed comparison

The governing distinction is between preservation of a complete quantum output and recovery from a particular accessible output. A subsequent classical observation is a third operation, and thermodynamic reset is a fourth question. This distinction is documented in Cheung's En07-A, En07-B, and Hawk04-B [CheungEn07A,CheungEn07B,CheungHawk04B].

The finite recovery structure used below is standard quantum error correction and no-hiding structure [KnillLaflamme,BraunsteinPati]. The approximate theorem uses the explicitly stated external input of Kretschmann, Schlingemann, and Werner [KSW]. The contribution here is a complete, typed comparison: the proofs, normalization, counterexamples, access restrictions, and endpoint error budget are stated in one place. No new microscopic law of black-hole evaporation is claimed.

What is taken from Hawking's paper

For a nonrotating uncharged black hole, Hawking's late-time packet calculation gives, in the natural-unit convention,

$$\langle n_{\omega \ell m} \rangle = \frac{\Gamma_{\omega \ell m}}{\exp\left(2\pi\omega/\kappa\right) - 1}$$

for bosons, with the corresponding plus sign in the denominator for fermions. Here *Γ_{ωℓm}* is the absorption probability, not an additional information-recovery certificate. The temperature is

$$T_H = \frac{\hbar \kappa}{2\pi k_{\mathrm{B}}},$$

when *κ* denotes the horizon redshift rate in inverse-time units. If surface gravity is instead expressed as an acceleration *κ_acc*, then *κ = κ_acc/c*. These are external semiclassical statements, not conclusions of the finite theorems below [Hawking1975].

Hawking's backreaction discussion associates net positive outgoing energy flux with decreasing black-hole mass and an ingoing negative-energy flux in the semiclassical description. Net mass loss requires the outgoing flux to exceed incoming energy. The same paper explicitly states that the quasi-stationary classical-metric approximation breaks down at the Planck-scale endpoint [Hawking1975]. Consequently, its final-disappearance discussion is not used here as a construction of a complete microscopic endpoint isometry.

The historical estimate that sufficiently light primordial black holes would have evaporated ``by now'' belongs to the cosmological age assumptions used in the 1975 paper. It is not a present-day numerical estimate in this manuscript.

The entropy expression relevant to the historical comparison is

$$\frac{S_{\mathrm{gen}}^{\mathrm{phys}}}{k_{\mathrm{B}}} = \frac{S_{\mathrm{out}}^{\mathrm{phys}}}{k_{\mathrm{B}}} + \frac{A}{4\ell_{\mathrm{P}}^2}, \qquad \ell_{\mathrm{P}}^2 = \frac{\hbar G}{c^3}.$$

The abbreviated expression *S + A/4* assumes the corresponding natural units. Black-hole mechanics and the generalized second law are separate gravitational and thermodynamic subjects [BardeenCarterHawking,Bekenstein1974,Hawking1975]. Neither an area theorem nor a generalized-second-law proof is derived from a code dimension in this paper.

Corrections retained throughout

• A thermal state, an occupation spectrum, or one prepared radiation density operator is not automatically a replacement *channel* on a quantum code. A channel statement must control all logical operators, including off-diagonal operators and external references.

• A verified nearly constant *complete accessible channel* obstructs recovery from that channel. In a complete isometric interface it also constrains possible complement-decoupling allowances. It is therefore inaccurate to say that such a channel estimate has no consequence for the complement. Thermal data alone, however, do not establish that channel estimate.

• Complete unitary evolution does not automatically conserve an independently chosen Hamiltonian. Energy conservation requires its own commutation or intertwining hypothesis.

• Complete disappearance is not promoted from a semiclassical extrapolation to a proved quantum-gravitational endpoint.

• The four obligations introduced in Definition specify this paper's same-model endpoint certificate. They are not a universal necessity theorem about every conceivable description of evaporation.

• ``XV-B not opened'' refers to the admission-dependent downstream quantum exit. It does not deny the existence of supplied technical revisions of XV-B that prove conditional lifting interfaces while leaving the inherited full map unresolved.

par **Release boundary** par The mathematical target is a declared finite quantum interface. No Bogoliubov calculation, gravitational factorization, bulk–boundary identification, coherent gravitational receiver, Page curve,

spacetime-emergence theorem, or microscopic evaporation endpoint is constructed. No statement below supplies the missing inherited characteristic quantum product.

par

The complete finite interface and error conventions

Definition. Let C, R, B be finite-dimensional Hilbert spaces, with $d = \dim C \geq 2$. A complete code interface is an isometry

$$V : C \to R \otimes B, \qquad V^\dagger V = I_C.$$

Its complete, accessible, and complementary channels are

$$\mathcal{V}(X) = VXV^\dagger, \qquad \mathcal{M}(X) = \mathrm{Tr}_B\,(VXV^\dagger), \qquad \mathcal{E}(X) = \mathrm{Tr}_R\,(VXV^\dagger).$$

The factor R is the declared quantum access. The factor B contains every other output of the complete prepared dilation, including preparation purifiers, apparatus environments, lost carriers, and retained memories.

The word ``complete'' is a modeling requirement, not a consequence of naming one environmental marginal. It is essential to Theorem and Example .

One way to obtain is to specify an encoding $J: C \to H_{payload}$, a fixed normalized ancilla $|\Omega\rangle$ containing any necessary purifiers, and a complete unitary U, and then set

$$V|\psi\rangle = U(J|\psi\rangle \otimes |\Omega\rangle).$$

The ancilla is fixed independently of the unknown logical input. The construction is mathematically isometric; whether it describes a gravitational preparation and evolution is a separate obligation.

For a linear map Λ between matrix spaces, its diamond norm is

$$\|\Lambda\|_\diamond = \sup_{m \geq 1} \sup_{\|X\|_1 = 1} \|(\mathrm{id}_m \otimes \Lambda)(X)\|_1.$$

For channels with a common input and output,

$$d_\diamond(\Phi, \Psi) = \frac{1}{2}\|\Phi - \Psi\|_\diamond, \qquad d_{\mathrm{tr}}(\rho, \sigma) = \frac{1}{2}\|\rho - \sigma\|_1.$$

For channel differences, the diamond norm can be evaluated on density operators with a reference of dimension equal to the input dimension. Thus the errors used here are worst-case reference-preserving errors, not errors for one selected input.

The finite stabilized norm facts in this convention are standard channel-norm facts; see the preliminaries of [KSW]. All distances between channels in this paper use .

Definition. For a density operator σ on a specified output space, define

$$\mathcal{T}_\sigma(X) = \mathrm{Tr}\,(X)\sigma.$$

Set

$$\varepsilon_{\mathrm{rec}}(\mathcal{M}) = \min_{\mathcal{D} : R \to C \text{ CPTP}} d_\diamond(\mathcal{D}\mathcal{M}, \mathrm{id}_C),$$
$$\delta_{\mathrm{leak}}(\mathcal{E}) = \min_{\sigma \in \mathrm{D}(B)} d_\diamond(\mathcal{E}, \mathcal{T}_\sigma).$$

Write

$$\tau_C = \frac{I_C}{d}, \qquad q_d = 1 - d^{-2}.$$

A decoder is deterministic: it is a completely positive, trace-preserving map on all of $L(R)$. Postselected success is not substituted for .

The minima exist because finite-dimensional state spaces and CPTP-map sets are compact, and the objectives are continuous.

Lemma. The following statements hold.

• CPTP pre- and postprocessing do not increase $d_{\diamond}$.

• For replacement channels with the same input,

$$d_{\diamond}(\mathscr{T}_{\sigma}, \mathscr{T}_{\omega}) = d_{\mathrm{tr}}(\sigma, \omega).$$

• If $V, W: C \to R \otimes B$ are isometries, then

$$d_{\diamond}(\mathrm{Ad}_V, \mathrm{Ad}_W) \le \min\{1, \|V - W\|_{\infty}\}.$$

The same bound holds for either of their two partial-trace channels.

• If $\mathscr{N}_1, \mathscr{N}_2$ have common interfaces, then

$$|\varepsilon_{\mathrm{rec}}(\mathscr{N}_1) - \varepsilon_{\mathrm{rec}}(\mathscr{N}_2)| \mathrm{led}_{\diamond}(\mathscr{N}_1, \mathscr{N}_2).$$

The analogous inequality holds for δ_{leak} of two channels with a common residual output.

Proof. A channel has diamond norm one, so submultiplicativity proves (i). For (ii), applying the replacement-channel difference to a referenced state ρ_{QC} gives $\rho_Q \otimes (\sigma - \omega)$. Its trace norm is $\|\sigma - \omega\|_1$.

For (iii), after adjoining any reference, expand

$$VXV^{\dagger} - WXW^{\dagger} = (V - W)XV^{\dagger} + WX(V^{\dagger} - W^{\dagger}).$$

The trace norm is at most $2\|V - W\|_{\infty}\|X\|_1$. Divide by two; channel distances are at most one. Part (i) gives the marginal statements.

For (iv), use any decoder for $\mathscr{N}_1$ also on $\mathscr{N}_2$, apply the triangle inequality and (i), minimize, and interchange the two channels. The leakage statement follows by using the same replacement state in both comparisons.

Proposition. Let $a: R \to R'$ and $b: B \to B'$ be isometries, and define

$$V' = (a \otimes b)V.$$

Then

$$\varepsilon_{\mathrm{rec}}(\mathrm{Tr}_{B'}\mathrm{Ad}_{V'}) = \varepsilon_{\mathrm{rec}}(\mathscr{N}), \qquad \delta_{\mathrm{leak}}(\mathrm{Tr}_{R'}\mathrm{Ad}_{V'}) = \delta_{\mathrm{leak}}(\mathscr{E}).$$

This statement does not authorize a joint relabeling that moves an inaccessible logical register into the accessible factor.

Proof. An isometry a has a CPTP left inverse

$$L_a(Y) = a^{\dagger}Ya + \mathrm{Tr}\,((I - aa^{\dagger})Y)\tau_R,$$

where τ_R is any fixed state on R. A decoder on R' restricts to one on R by composition with Ad_a; a decoder on R extends to R' by composition with L_a. These two operations give the recovery equality.

For leakage, embedding a replacement state by b gives one inequality. Applying L_b to an arbitrary candidate replacement state on B' gives the other inequality by contraction. A joint transformation mixing R and B is not of the stated product form and need not preserve either optimization problem.

Complete restoration and exact outgoing recovery

Theorem. For the interface , the complete channel $\mathscr{V}$ has the CPTP left inverse

$$\mathscr{L}_V(Y) = V^{\dagger}YV + \mathrm{Tr}\,((I_{RB} - VV^{\dagger})Y)\tau_C.$$

Moreover, for every reference Q and every operator X on $Q \otimes C$,

$$\left\|(I_Q \otimes V)X(I_Q \otimes V^{\dagger})\right\|_1 = \|X\|_1.$$

The left inverse requires access to the joint output RB.

Proof. The compression term in is completely positive. The other term is a positive-effect measurement followed by preparation and is completely positive. Their traces sum to $Tr\ Y$, so $\mathscr{L}_V$ is trace preserving. On $VXV^{\dagger}$, the second term vanishes and the first returns X. An isometric embedding preserves all nonzero singular values, including after tensoring with a reference identity. This proves .

This complete-output restoration is also documented in

[CheungEn07A,CheungEn07B,CheungEn04A,CheungHawk04A]. It does not give a decoder on R alone.

Fix a code basis $\{|i\rangle\}_{i=1}^{d}$ and let

$$E_{ij} = |i\rangle\langle j|, \qquad |\Phi_d\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |i\rangle_Q |i\rangle_C.$$

The normalized Choi state of a channel Φ with input C is

$$J_\Phi = (\mathrm{id}_Q \otimes \Phi)(|\Phi_d\rangle\langle\Phi_d|) = \frac{1}{d} \sum_{i,j} E_{ij}^Q \otimes \Phi(E_{ij}).$$

Theorem. For $V: C \to R \otimes B$, the following conditions are equivalent.

• A deterministic CPTP decoder satisfies $\mathcal{D}\mathcal{N} = id_C$.

• There is one state σ_B such that $\mathcal{E} = \mathcal{T}_{\sigma_B}$.

• For every logical matrix unit, $\mathcal{E}(E_{ij}) = \delta_{ij}\sigma_B$.

• The normalized complementary Choi state satisfies

$$J_{\mathcal{E}} = \frac{I_Q}{d} \otimes \sigma_B.$$

• For every residual observable $O \in L(B)$,

$$V^\dagger (I_R \otimes O) V = \mathrm{Tr}\,(O\sigma_B) I_C.$$

If these conditions hold and $r = rank\ \sigma_B$, there are a Hilbert space K of dimension r, a state σ_K, and an isometry $A: C \otimes K \to R$ such that

$$\mathcal{N}(X) = A(X \otimes \sigma_K) A^\dagger.$$

An exact decoder on all of $L(R)$ is

$$\mathcal{D}(Y) = \mathrm{Tr}_K\,(A^\dagger Y A) + \mathrm{Tr}\,((I_R - AA^\dagger)Y)\tau_C.$$

Every external reference is preserved.

Proof. Matrix units span $L(C)$, so (ii) and (iii) are equivalent. Expansion of gives equivalence with (iv). Trace duality gives

$$\mathrm{Tr}\,(O\mathcal{E}(X)) = \mathrm{Tr}\left(V^\dagger (I_R \otimes O) V X\right),$$

which proves equivalence with (v).

Assume (ii), and diagonalize

$$\sigma_B = \sum_{\alpha=1}^{r} p_\alpha |b_\alpha\rangle\langle b_\alpha|, \qquad p_\alpha > 0, \quad \sum_\alpha p_\alpha = 1.$$

The residual support condition implies

$$V|i\rangle = \sum_{\alpha=1}^{r} \sqrt{p_\alpha}\, |r_{i\alpha}\rangle_R |b_\alpha\rangle_B.$$

The coefficients of $\mathcal{E}(E_{ij})$ give

$$\langle r_{j\beta} \mid r_{i\alpha}\rangle = \delta_{ij}\delta_{\alpha\beta}.$$

Thus the rule $A(|i\rangle \otimes |\alpha\rangle) = |r_{i\alpha}\rangle$ defines an isometry, and $\sigma_K = \sum_{\alpha} p_{\alpha} |\alpha\rangle\langle\alpha|$ gives . The two terms in are completely positive; their traces sum to $Tr\ Y$. Substitution of proves (i).

Conversely, assume (i). A finite Stinespring dilation of the decoder has an isometry $L: R \to C \otimes G$ [Stinespring]. Define

$$Z = (L \otimes I_B) V : C \to C \otimes G \otimes B.$$

The decoded output on input $|i\rangle$ is pure, hence

$$Z|i\rangle = |i\rangle \otimes |\eta_i\rangle_{GB}$$

for normalized η_i. Exact recovery of E_ij implies $\langle \eta_j \mid \eta_i \rangle = 1$. All η_i therefore equal one vector η. Tracing C and G gives

$$\mathscr{E}(E_{ij}) = \delta_{ij} \mathrm{Tr}_G \, |\eta\rangle\langle\eta|,$$

which is (iii).

Equality of channels on all logical matrix units persists after tensoring with any reference identity. This proves reference preservation.

Theorem is the exact correction/no-hiding structure associated with [KnillLaflamme,BraunsteinPati]. The explicit complete-complement and decoder formulations occur in the supplied Cheung manuscripts [CheungEn07A,CheungEn07B,CheungHawk04A,CheungHawk03A]. Their physical models are not identified with one another here.

Corollary. Exact recovery implies

$$\dim R \geq d \, \mathrm{rank} \, \sigma_B.$$

If σ_B is pure, $\mathscr{N}$is an isometric encoding. Equation is not an area law.

Proof. The dr vectors $r_\{i\alpha\}$ constructed in the proof of Theorem are orthonormal in R. If $r = 1$, reduces to conjugation by an isometry $C \to R$. No geometric area or gravitational dynamics enters this count.

Corollary. If $\mathscr{N}$is constant on the entire code, the complete output B exactly recovers the code. If $\mathscr{E}$ is constant, R exactly recovers it. For $d \geq 2$, both complementary outputs cannot separately recover the entire unknown code, and both cannot be constant.

Proof. Interchange R and B in Theorem . An exactly recoverable output has constant complement. A constant channel cannot recover two distinct logical states.

Destructive Enter, forgetting, and classical records

The measured factor in this section is explicitly B. A classical record is denoted by Z, so it is not confused with the quantum access factor R.

Definition. Choose an orthonormal basis $\{|z\rangle\}$ of B, set

$$M_z = I_R \otimes \langle z|_B : R \otimes B \to R,$$

and define

$$\mathrm{Enter}_B(X) = \sum_z M_z X M_z^\dagger \otimes |z\rangle\langle z|_Z, \qquad \mathrm{Enter}_B : \mathrm{L}(R \otimes B) \to \mathrm{L}(R \otimes Z).$$

The record Z is classical: the output is diagonal in its declared basis. The measured quantum factor B is not returned. Forgetting the record is $Drop_Z = Tr_Z$.

This is the destructive primitive, not a nondestructive instrument that returns the measured register. The distinction is documented in [CheungEn03A,CheungEn07A,CheungEn07B,CheungHawk04B].

Lemma. On all operators, including all referenced operators,

$$\mathrm{Drop}_Z \circ \mathrm{Enter}_B = \mathrm{Tr}_B.$$

A complete coherent dilation is the isometry

$$W(|r\rangle_R |z\rangle_B) = |r\rangle_R |z\rangle_Z |z\rangle_F.$$

It satisfies

$$W^\dagger W = I_{RB}, \qquad \mathrm{Tr}_F \mathrm{Ad}_W = \mathrm{Enter}_B.$$

Proof. Write

$$X = \sum_{z,z'} X_{zz'} \otimes |z\rangle\langle z'|_B.$$

Then

$$\mathrm{Enter}_B(X) = \sum_z X_{zz} \otimes |z\rangle\langle z|_Z, \qquad \mathrm{Drop}_Z\mathrm{Enter}_B(X) = \sum_z X_{zz} = \mathrm{Tr}_B X.$$

The same block expansion applies with an untouched reference. The vectors on the right side of preserve inner products of basis vectors, so W is an isometry. Tracing F removes terms with $z \neq z^{\{'\}}$ and gives .

Proposition. Equation does not assert physical annihilation of a state or its complete information. The complete dilation preserves the unknown state and its references, whereas the restricted output after forgetting need not recover it. It also does not determine a heat, work, or memory-reset cost.

Proof. Theorem applies to W and gives a complete-output left inverse. Restriction by partial trace changes the available output but not the isometry identity. No Hamiltonian, bath temperature, work reservoir, or resource-return condition occurs in . The thermodynamic non-identification is made explicit in Proposition .

For example, the coherent image of a superposition is

$$W\Big(\sum_z \psi_z |z\rangle_B\Big) = \sum_z \psi_z |z\rangle_Z |z\rangle_F,$$

suppressing a spectator R. It is not two independent copies of that unknown superposition.

Theorem. Let $\mathscr{M}: C \to Z$ be any channel with a purely classical output, and let $\mathscr{P}: Z \to C$ be any deterministic CPTP decoder. Then

$$d_\diamond(\mathscr{P}\mathscr{M}, \mathrm{id}_C) \geq 1 - d^{-1}.$$

The conclusion concerns record-only processing. It does not apply to an instrument that also retains an authorized quantum output.

Proof. A finite classical-output channel has the form

$$\mathscr{M}(X) = \sum_z \mathrm{Tr}\,(F_z X)|z\rangle\langle z|, \qquad F_z \geq 0, \quad \sum_z F_z = I_C.$$

Only $\mathscr{R}(|z\rangle\langle z|) = \rho_z$ is relevant on this output. Consequently

$$\mathscr{P}\mathscr{M}(X) = \sum_z \mathrm{Tr}\,(F_z X)\rho_z$$

is measure-and-prepare. Its Bell-input state is separable across $Q \mid C$.

For a normalized product vector $|a\rangle_Q|b\rangle_C$,

$$|\langle \Phi_d \mid a, b\rangle|^2 \leq d^{-1}$$

by Cauchy–Schwarz. Convexity gives the same Bell overlap bound for every separable state. Testing the projector $|\Phi_d\rangle\langle\Phi_d|$ therefore gives trace distance at least $1 - d^{\{-1\}}$ from the ideal Bell output. This test is one admissible referenced channel test, proving .

A measurement of outgoing Hawking modes is thus a channel applied to quantum access; it is not itself a quantum-return decoder. A useful record may coexist with quantum return when the authorized quantum factor is retained. A record-only replacement cannot preserve arbitrary reference entanglement.

Featureless channels and the thermal-state inference gap

Lemma. For every replacement channel with input dimension d,

$$\varepsilon_{\mathrm{rec}}(\mathscr{T}_\omega) = q_d = 1 - d^{-2}.$$

For every channel with input C, $\varepsilon_{rec} \leq_d$.

Proof. Any decoder applied to a replacement channel produces another replacement channel $\mathscr{T}_{\tau}$ on C. Its Bell-input output is $I_Q/d \otimes \tau$, whose Bell overlap is $d^{\{-2\}}$. The Bell-projector test therefore gives error at least q_d.

For attainment, use the decoder that always prepares τ_C. Let U_{ab}, $0 \leq a, b < d$, be the Weyl unitaries. Their uniform twirl is $\mathscr{T}_{\tau_C}$, so

$$\mathrm{id}_C - \mathscr{T}_{\tau_C} = (1 - d^{-2})\mathrm{id}_C - d^{-2} \sum_{(a,b) \neq (0,0)} \mathrm{Ad}_{U_{ab}}.$$

The triangle inequality and diamond norm one of each channel give half-diamond norm at most *q_d*. The Bell test gives equality. The same constant decoder can be used after any channel, proving the final assertion.

Proposition. Suppose

$$d_{\diamond}(\mathcal{N}, \mathcal{T}_{\omega}) \le \eta.$$

Every deterministic decoder obeys

$$d_{\diamond}(\mathcal{D}\mathcal{N}, \mathrm{id}_C) \ge \max\{0, q_d - \eta\}.$$

This is a recovery *lower* bound.

Proof. Contraction gives *d_{◇}(𝒟𝒩, 𝒟𝒯_{ω}) ≤ η*. The triangle inequality and the replacement-channel lower bound in Lemma give

$$d_{\diamond}(\mathcal{D}\mathcal{N}, \mathrm{id}_C) \ge_d -\eta.$$

Distances are nonnegative.

The corresponding comparisons in [CheungEn07A,CheungEn07B,CheungHawk04A] must be read in this direction. Closeness of an accessible channel to a replacement is not a recovery upper certificate.

A finite thermal comparison state

For a prescribed finite Hamiltonian *H_R* and temperature *T > 0*, one may define

$$\omega_T = \frac{\exp(-H_R/(k_{\mathrm{B}}T))}{\mathrm{Trexp}(-H_R/(k_{\mathrm{B}}T))}.$$

Proposition applies if a model proves *d_{◇}(𝒩, 𝒯_{ω_T}) ≤ η*. Defining , or matching a selected occupation law to it, does not prove this hypothesis.

In particular, is an occupation statement for specified semiclassical preparation and mode data. It is not a uniform half-diamond estimate on an independently declared microscopic code. Absorption factors, packet selection, and correlations also prevent replacing the entire outgoing continuum by an unqualified finite Gibbs density.

A finite selection of bosonic modes still has an infinite occupation space. Applying the finite theorems requires a specified finite encoding or cutoff and a reference-preserving allowance for the discarded sector. Such an allowance must not be silently omitted from the complete residual. Finite-packet and regulated comparison interfaces are documented in [CheungEn02E,CheungHawk02E]; they are not imported as gravitational constructions.

Example. Let *R = C*, choose any full-rank Gibbs state *ω* on *C*, and compare *𝒩_1 = id_C* with *𝒩_2 = 𝒯_{ω}*. For the prepared input *ω*,

$$\mathcal{N}_1(\omega) = \mathcal{N}_2(\omega) = \omega.$$

Nevertheless

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_1) = 0, \qquad \varepsilon_{\mathrm{rec}}(\mathcal{N}_2) = q_d.$$

Both channels admit complete finite isometric dilations. Thus equality of the entire accessible state for one preparation, not merely equality of a spectrum, fails to determine recovery.

Proof. The recovery values follow from identity recovery and Lemma . A dilation of *id_C* is *|ψ⟩ ↦ |ψ⟩_R|0⟩_B*. A dilation of *𝒯_{ω}* is *|ψ⟩ ↦ |χ_{ω}⟩_RP|ψ⟩_L* with *B = P ⊗ L*, where *χ_{ω}* purifies *ω*.

Remark. A proposed uniform thermal replacement comparison must specify which logical inputs are compared. If their macroscopic mass, angular momentum, charge, preparation, or access changes, a common replacement state is an additional claim. Holding those data fixed is a possible model specification, not a conclusion of the finite-interface theorem.

Approximate complement certificates and finite tests

We import the finite-dimensional information–disturbance consequence of the Stinespring continuity theorem [KSW]. For an isometric channel *𝒩* with its complete complement *ℰ*, define full-diamond quantities

$$a = \min_{\mathscr{D}} \| \mathscr{D}\mathscr{N} - \mathrm{id}_C \|_\diamond, \qquad b = \min_{\sigma} \| \mathscr{E} - \mathscr{T}_\sigma \|_\diamond.$$

The imported estimates are

$$\frac{1}{4}a^2 \le b, \qquad b \le 2\sqrt{a}.$$

Ancilla enlargement in a Stinespring proof is mathematical dilation bookkeeping. It is not permission to give inaccessible physical factors to the decoder.

No companion Cheung manuscript is needed as a premise for this external theorem. Its half-diamond use is also documented in [CheungEn07B,CheungHawk01B,CheungHawk03A].

Theorem. Under Convention ,

$$\varepsilon_{\mathrm{rec}}(\mathscr{N}) \le \min\{q_d, \sqrt{2\delta_{\mathrm{leak}}(\mathscr{E})}\},$$
$$\delta_{\mathrm{leak}}(\mathscr{E}) \le \min\{1, \sqrt{2\varepsilon_{\mathrm{rec}}(\mathscr{N})}\}.$$

In particular, if a specified state satisfies $d_{\diamond}(\mathscr{E}, \mathscr{T}_{\sigma}) \le \zeta$, then a deterministic decoder exists with error at most

$$\min\{q_d, \sqrt{2\zeta}\}.$$

Also,

$$\varepsilon_{\mathrm{rec}}(\mathscr{N}) \ge \frac{1}{2}\delta_{\mathrm{leak}}(\mathscr{E})^2.$$

Proof. By the definitions, $a = 2\varepsilon_{rec}(\mathscr{N})$ and $b = 2\delta_{leak}(\mathscr{E})$. The first inequality of becomes $\varepsilon_{rec}^2 \le 2\delta_{leak}$. The second becomes $\delta_{leak} \le \sqrt{(2\varepsilon_{rec})}$. Lemma gives the independent upper cap $\varepsilon_{rec} \le_d$, and channel distances give $\delta_{leak} \le 1$. A specified replacement allowance upper-bounds the minimized leakage. Squaring proves .

A bound on $d_{\diamond}(\mathscr{E}, \mathscr{T}_{\sigma})$ is a recovery *upper* certificate. A bound on $d_{\diamond}(\mathscr{N}, \mathscr{T}_{\omega})$ is a recovery *lower* certificate. The output factor, the replacement state, the quantifier over code inputs, and the factor of one half in are part of the statement.

Corollary. If one complete interface satisfies

$$d_\diamond(\mathscr{N}, \mathscr{T}_\omega) \le \eta, \qquad d_\diamond(\mathscr{E}, \mathscr{T}_\sigma) \le \zeta,$$

then

$$q_d \le \eta + \sqrt{2\zeta}.$$

Equivalently,

$$\zeta \ge \frac{1}{2}(\max\{0, q_d - \eta\})^2.$$

In particular, exact accessible constancy and exact complement constancy cannot coexist for $d \ge 2$.

Proof. Proposition lower-bounds the optimized recovery error, while Theorem upper-bounds it. If $\eta < q_d$, combine the two inequalities; if $\eta \ge_d$, is immediate. The exact conclusion also follows without KSW from Corollary .

This is the precise correction to the phrase ``thermal implies information loss." Thermal data alone do not give . If the complete channel hypothesis is actually proved, it gives a rigorous obstruction and a complement-allowance constraint.

Finite matrix certificates

Lemma. For channels Φ, Ψ with common input dimension d,

$$d_{\mathrm{tr}}(J_\Phi, J_\Psi) \le \mathrm{led}_\diamond(\Phi, \Psi) \le \min\{1, d\, d_{\mathrm{tr}}(J_\Phi, J_\Psi)\}.$$

If $\|\Phi(E_{ij}) - \Psi(E_{ij})\|_1 \le r$ for every matrix unit, then

$$d_\diamond(\Phi, \Psi) \le \min\{1, dr/2\}.$$

Proof. The Bell state is an admissible referenced input, proving the first inequality. A pure referenced input can be written, after a reference isometry, as $(A \otimes I_C)|\Phi_d\rangle$ with $\|A\|_{\infty}^2 \le d$. Applying $\Phi - \Psi$ and multiplying its Choi difference on the reference side gives a trace norm at most $d\|J_{\Phi} - J_{\Psi}\|_1$. Convexity and the channel-difference state optimization prove the upper bound.

Alternatively, write a pure referenced input vector as $\sum_i |v_i\rangle \otimes |i\rangle$, with $\sum_i \|v_i\|^2 = 1$. The matrix-unit hypothesis bounds the output difference by

$$r\sum_{i,j} \|v_i\|\|v_j\| \le dr.$$

Divide by two and use convexity.

These conversions are finite-input estimates, not dimension-independent replacements for a channel certificate. Related fixed-model certificates are documented in [CheungEn07B,CheungEm03H,CheungHawk02E].

Proposition. Let $\sigma \in D(B)$ and suppose

$$\chi = d_{\mathrm{tr}}(J_{\mathscr{E}}, I_Q/d \otimes \sigma).$$

Then

$$\varepsilon_{\mathrm{rec}}(\mathscr{N}) \le \min\{q_d, \sqrt{2\min\{1, d\chi\}}\}.$$

If a justified model approximation satisfies $d_{\diamond}(\mathscr{E}_{phys}, \mathscr{E}) \le \lambda$ on the same complete residual interface, then

$$\delta_{\mathrm{leak}}(\mathscr{E}_{\mathrm{phys}}) \le \min\{1, \lambda + d\chi\}.$$

A physical recovery conclusion additionally requires $\mathscr{E}_{phys}$ to be the complete complement of the declared physical accessible channel.

Proof. Use Lemma with $J_{\mathscr{T}_{\sigma}} = I_Q/d \otimes \sigma$, then apply Theorem . For the physical comparison, use the triangle inequality. The final sentence retains the hypothesis needed to apply that theorem to the physical channel.

Lemma. If two logical states satisfy

$$x = d_{\mathrm{tr}}(\mathscr{E}(\rho_0), \mathscr{E}(\rho_1)),$$

then

$$\delta_{\mathrm{leak}}(\mathscr{E}) \ge x/2, \qquad \varepsilon_{\mathrm{rec}}(\mathscr{N}) \ge x^2/8.$$

If $O = O^{\dagger}$, $\|O\|_{\infty} \le 1$, and

$$w = |\mathrm{Tr}\,(O\mathscr{E}(\rho_0 - \rho_1))|,$$

then

$$\delta_{\mathrm{leak}}(\mathscr{E}) \ge w/4, \qquad \varepsilon_{\mathrm{rec}}(\mathscr{N}) \ge w^2/32.$$

Proof. For every replacement state σ,

$$x \le d_{\mathrm{tr}}(\mathscr{E}(\rho_0), \sigma) + d_{\mathrm{tr}}(\sigma, \mathscr{E}(\rho_1)) \le 2d_{\diamond}(\mathscr{E}, \mathscr{T}_{\sigma}).$$

Minimize and use . Trace duality gives $w \le 2x$, proving the remaining statements.

A positive residual witness establishes dependence on some logical input. It does not establish residual-only recovery of the entire code.

Why the residual must include purifiers

Example. Let $C = R = \mathbb{C}^2$, and let E and P have bases indexed by $a = (a_1, a_2) \in \{0, 1\}^2$. Prepare

$$|\Omega\rangle_{EP} = \frac{1}{2}\sum_a |a\rangle_E|a\rangle_P.$$

Let $U_a = X^{a_1}Z^{a_2}$ be the four Pauli representatives and apply the controlled unitary to C and E. The complete isometry is

$$V|\psi\rangle = \frac{1}{2}\sum_a U_a|\psi\rangle_R|a\rangle_E|a\rangle_P, \qquad B = E \otimes P.$$

Then

$$\mathcal{M}(X) = \mathrm{Tr}\,(X) I_R/2,$$
$$\mathrm{Tr}_P\,\mathcal{E}(X) = \mathrm{Tr}\,(X) I_E/4,$$
$$\mathcal{E}(X) = \frac{1}{4}\sum_{a,b} \mathrm{Tr}\,(U_b^\dagger U_a X)|a,a\rangle\langle b,b|.$$

The selected residual E is constant, but the complete residual EP is not. The complete residual exactly recovers the unknown qubit.

Proof. Orthogonality of the key labels and unitarity of U_a show $V^{\dagger}V = I_C$. Tracing EP gives the Pauli twirl, which is . Tracing P first removes off-diagonal key terms; tracing R then gives . Tracing only R gives . For example, an off-diagonal key coefficient with $U_b^{\dagger}U_a = Z$ distinguishes the two Z eigenstates. Finally, and Corollary give exact recovery from EP.

The preparation is a purification of a mixed key. Removing P from the ledger is precisely the error that would falsely suggest two constant outputs in a complete pure dilation. This example does not identify P with a black-hole interior, a holographic boundary, or a gravitational partner algebra.

Entropy, energy, erasure cost, and leakage

Selected entropy and spectral data

All von Neumann entropies in the finite arguments use natural logarithms and are dimensionless: $S(\rho) = -Tr\,(\rho log\,\rho)$. A thermodynamic entropy in conventional units includes the factor k_B.

Proposition. Choose any finite state σ_K, a purification $|\chi\rangle_{KP}$, and a logical register $L \simeq C$. Let

$$R = C_R \otimes K, \qquad B = P \otimes L.$$

Define two complete isometries, with tensor factors ordered as R and B, by

$$V_{\mathrm{return}}|\psi\rangle = |\psi\rangle_{C_R}|\chi\rangle_{KP}|0\rangle_L,$$
$$V_{\mathrm{retain}}|\psi\rangle = |0\rangle_{C_R}|\chi\rangle_{KP}|\psi\rangle_L.$$

Their accessible channels are

$$\mathcal{N}_{\mathrm{return}}(X) = X \otimes \sigma_K, \qquad \mathcal{N}_{\mathrm{retain}}(X) = \mathrm{Tr}\,(X)|0\rangle\langle 0|_{C_R} \otimes \sigma_K.$$

For every pure logical input, the two accessible outputs have the same eigenvalues and the same entropy $S(\sigma_K)$. For the input $|0\rangle\langle 0|$, the entire accessible density operators are equal. Nevertheless

$$\varepsilon_{\mathrm{rec}}(\mathcal{N}_{\mathrm{return}}) = 0, \qquad \varepsilon_{\mathrm{rec}}(\mathcal{N}_{\mathrm{retain}}) = q_d.$$

If $H_R = I_{C_R} \otimes H_K$, every energy-measurement distribution is also equal in the two models for all logical inputs.

Proof. Both maps preserve logical inner products. The partial traces give the displayed channels. For a pure $|\psi\rangle$, both accessible states are a rank-one logical state tensored with σ_K; therefore their nonzero spectra and entropies agree. The chosen input $|0\rangle$ makes the density operators identical. Tracing K exactly decodes the return model. The retain model is a replacement channel, so Lemma gives its error. A spectral effect of $H_R = I \otimes H_K$ acts as the identity on the logical factor and has the same expectation in both outputs.

Taking σ_K to be a Gibbs state makes the selected energy statistics thermal. It does not make the entire return channel constant. Thus selected entropy, occupation calibration, and logical recovery answer different questions. Sequential versions of this separation are documented in [CheungHawk01B]; finite occupation and source-channel comparisons are documented in [CheungEm03E,CheungEm03F,CheungEm03G].

The complete energy ledger

Definition. For a complete unitary U on a finite system with declared Hamiltonian H_{tot}, exact conservation means

$$U^\dagger H_{\mathrm{tot}} U = H_{\mathrm{tot}}.$$

For an isometry between declared input and output energy spaces, the stronger intertwining form is

$$H_{\mathrm{out}}V = VH_{\mathrm{in}}.$$

These are additional hypotheses.

Proposition. Equation preserves the expectation of *H_tot* for every input state. Equation preserves the corresponding input energy expectation. Neither property follows merely from *U^{†}U = I* or *V^{†}V = I*. Neither property by itself implies outgoing recovery.

Proof. The first two statements follow by trace duality. For the nonimplication, a qubit bit flip is unitary but does not commute with $H = \varepsilon|1\rangle\langle 1|$ when $\varepsilon > 0$. Failure of recovery despite exact conserving evolution is proved in Proposition .

Free-Hamiltonian conservation, as in the finite comparison documented in [CheungEm03E], is not automatically a ledger for switching work, interaction energy, or reusable apparatus reset.

A thermal implementation requires additional data

Definition. A thermodynamic cost claim must specify at least:

• the physical memory and its initial state or input ensemble;

• all relevant Hamiltonians and interaction/switching conventions;

• reservoirs, their initial states and temperatures;

• work storage, controllers, and usable side information;

• the complete dynamics and final correlations;

• definitions of heat and work, and conditions for returning reusable resources. A channel on selected visible factors is not this package.

Landauer's original work concerns physical implementation [Landauer1961]. A precise finite thermal equality is available under additional assumptions [ReebWolf]; it is not inferred from .

Proposition. Let a memory M and reservoir L initially have the product state $\rho_M \otimes \gamma_L$, where

$$\gamma_L = e^{-\beta H_L}/\mathrm{Tr}\, e^{-\beta H_L}, \qquad \beta = (k_{\mathrm{B}}T)^{-1} > 0.$$

After a specified unitary on ML, write the state as $\rho_{ML}^{\{'\}}$ and define

$$Q_L = \mathrm{Tr}\,(H_L(\rho_L^{'} - \gamma_L)), \qquad \Delta S_M = S(\rho_M) - S(\rho_M^{'}).$$

Then

$$\beta Q_L = \Delta S_M + I(M:L)_{\rho'} + D(\rho_L^{'}\|\gamma_L) \geq \Delta S_M.$$

Here D is quantum relative entropy. This proposition neither constructs a reset operation nor optimizes its work or cyclic resource cost.

Proof. Unitary entropy preservation and the product preparation give

$$I(M:L)_{\rho'} = S(\rho_M^{'}) + S(\rho_L^{'}) - S(\rho_M) - S(\gamma_L).$$

Hence $S(\rho_L^{\{'\}}) - S(\gamma_L) = \Delta S_M + I(M:L)_{\rho^{\{'\}}}$. Using $\log \gamma_L = -\beta H_L - \log Z$ gives

$$D(\rho_L^{'}\|\gamma_L) = \beta Q_L - (S(\rho_L^{'}) - S(\gamma_L)).$$

Combining the identities proves the equality. Mutual information and relative entropy are nonnegative. The operation and its resource constraints are hypotheses, not outputs of the calculation.

Equation is the finite entropy-balance identity of [ReebWolf], included with its assumptions and a local derivation. It is not a gravitational generalized second law.

Proposition. Equality of a selected visible channel does not determine a unique reservoir heat, work-storage change, or reusable-memory reset ledger, even when the appended complete dynamics conserves a specified total energy.

Proof. Take any fixed visible implementation and append independent qubits W, L with

$$H_W = H_L = \varepsilon|1\rangle\langle 1|, \qquad \varepsilon > 0.$$

Prepare W in $|1\rangle$ and L in its Gibbs state $\gamma_L = diag\,(1 - p, p)$, with $0 < p < 1$. Compare doing nothing on WL with swapping W and L. Both spectator operations conserve $H_W + H_L$, and neither changes the selected visible channel.

The first operation has $Q_L = 0$. The swap leaves L excited and has $Q_L = \varepsilon(1 - p) > 0$. The battery's energy decreases by the same amount. The two implementations therefore have different heat and resource-return ledgers despite the same visible channel. The swap consumes a resource; it is not claimed to be a cyclic cost optimum. Precisely that missing resource-restoration specification prevents a reset-cost inference from channel equality.

par **The three ledgers** par Complete energy conservation is a statement about specified Hamiltonians and complete dynamics. Thermodynamic erasure cost is a statement about a specified implementation and resource restoration. Residual leakage is the dimensionless channel distance . None is identified with either of the others. In particular, leakage is not heat, stress–energy, horizon area, or geometric backreaction.

par

Access-preserving endpoint certificates

The endpoint statements below compare maps on the same code and retain the distinction between accessible and residual factors. They do not obtain an endpoint by merely declaring that a mass ledger has reached zero.

Definition. A same-model endpoint package for the declared code must supply:

• **Complete preparation.** A justified code and a complete pure prepared isometry $P_0: C \to H_{tot}$. Mixed ancillary preparations are purified and their purifiers remain in the complete system.

• **Complete evolution.** A specified logical-input-independent unitary family U_t on the complete system, on a justified time domain. Any energy-conservation claim has the separate hypotheses of Definition .

• **Justified access and endpoint comparison.** Complete output identifications $F_t: H_{tot} \to R_t \otimes B_t$ and factorwise isometries

$$a_t : R_t \to R_*, \qquad b_t : B_t \to B_*$$

into common finite comparison spaces, preserving the declared access roles. Define

$$\widehat{V}_t = (a_t \otimes b_t)F_tU_tP_0.$$

There must be an endpoint isometry $V_{*}: C \to R_{*} \otimes B_{*}$ and a proved allowance

$$\nu_t = \min\{1, \left\|\widehat{V}_t - V_*\right\|_\infty\} \to 0.$$

The comparison maps are not allowed to move inaccessible logical registers into R_{*}.

• **Complete complement allowance.** For $\mathscr{E}_{*} = Tr_{R_{*}} Ad_{V_{*}}$, a state $\sigma_{*} \in D(B_{*})$ and a proved bound

$$d_\diamond(\mathscr{E}_*, \mathscr{T}_{\sigma_*}) \le \zeta.$$

For a physical use of the package, all identifications and estimates must be justified within one microscopic model, on compatible domains and in a common regime of validity.

Definition gives sufficient data for the theorem. It does not derive any of them from the outgoing occupation law. It is the finite form of the conditional endpoint framing documented in [CheungEn07B,CheungHawk04B].

The raw and embedded access channels are

$$\mathscr{N}_t^{\mathrm{raw}} = \mathrm{Tr}_{B_t}\mathrm{Ad}_{F_tU_tP_0}, \qquad \widehat{\mathscr{N}}_t = \mathrm{Ad}_{a_t}\mathscr{N}_t^{\mathrm{raw}}.$$

Similarly, $\mathscr{E}_t = Tr_{R_{*}} Ad_{V_t}$.

Theorem. Suppose Definition is satisfied. Then

$$d_\diamond(\widehat{\mathscr{N}}_t, \mathscr{N}_*) \le \nu_t,$$

$$d_\diamond(\widehat{\mathscr{E}}_t, \mathscr{E}_*) \le \nu_t,$$

$$\delta_{\mathrm{leak}}(\widehat{\mathscr{E}}_t) \le \min\{1, \zeta + \nu_t\}.$$

Under the KSW input,

$$\varepsilon_{\mathrm{rec}}(\mathscr{N}_t^{\mathrm{raw}}) \le \min\{q_d,\ \nu_t + \sqrt{2\zeta},\ \sqrt{2\min\{1, \zeta + \nu_t\}}\}.$$

There is an endpoint-optimal deterministic decoder $\mathscr{D}_\{*\}$ such that the fixed decoder $\mathscr{D}_\{*\}Ad_a_t$ on the raw access has error at most

$$\min\{1, \nu_t + \varepsilon_{\mathrm{rec}}(\mathscr{N}_*)\}.$$

If $\zeta = 0$, the endpoint recovery is exact and can use . If $\sigma_\{*\}$ is pure in that exact case, the endpoint radiation channel is an isometric encoding.

Proof. Every map $\mathscr{V}_t$ is isometric because it is a composition of isometries and a unitary. The first two inequalities follow from Lemma . The triangle inequality with gives .

Choose an optimal endpoint decoder, which exists by compactness. Contraction and the triangle inequality give

$$d_\diamond(\mathscr{D}_* \widehat{\mathscr{N}}_t, \mathrm{id}_C) \le \nu_t + \varepsilon_{\mathrm{rec}}(\mathscr{N}_*),$$

proving . Theorem gives $\varepsilon_rec(\mathscr{N}_\{*\}) \le \sqrt{(2\zeta)}$. Alternatively apply that theorem directly to . The constant decoder gives the independent cap q_d. Proposition identifies the optimized raw and embedded recovery errors and proves .

For $\zeta = 0$, is exact channel equality. Theorem and Corollary give the remaining conclusions.

Corollary. Suppose an actual quantum-output receiver gives a channel $\mathscr{N}_t\text{^}recv: C \to R_\{*\}$ with a proved allowance

$$d_\diamond(\mathscr{N}_t^{\mathrm{recv}}, \widehat{\mathscr{N}}_t) \le \alpha_t.$$

Then

$$\varepsilon_{\mathrm{rec}}(\mathscr{N}_t^{\mathrm{recv}}) \le \min\{q_d, \alpha_t + \nu_t + \sqrt{2\zeta}\}.$$

A specified fixed endpoint decoder has error at most $\min\{1, \alpha_t + \nu_t + \varepsilon_rec(\mathscr{N}_\{*\})\}$. The receiver assumption is not a consequence of an observable probability comparison or a classical count record.

Proof. Apply contraction and the triangle inequality once more to the fixed decoder in Theorem . Use the endpoint upper bound and the independent constant-decoder cap. The final sentence distinguishes the stated quantum-output channel hypothesis from weaker data.

The coherent-access distinction is important in the documentary comparisons [CheungEm03D,CheungEm03G]. A decoder on an abstract packet factor does not construct a local field-to-memory receiver.

Corollary. If, in addition to the endpoint package,

$$d_\diamond(\widehat{\mathscr{N}}_t, \mathscr{T}_{\omega_t}) \le \eta_t,$$

then

$$q_d \le \eta_t + \nu_t + \sqrt{2\zeta}.$$

Thus a nearly featureless complete accessible channel and a small outgoing-recovery endpoint budget cannot be independently asserted in the same interface when these allowances violate .

Proof. Proposition gives $\varepsilon_rec(\mathscr{N}_t) \ge_d -\eta_t$. Theorem gives the upper bound $\nu_t + \sqrt{(2\zeta)}$. Combine them.

Corollary. If one physical gravitational model supplies the four obligations with justified outgoing quantum access, then the endpoint and finite-time conclusions of Theorem apply to its declared finite code and preserve external references. If a physical receiver comparison is also supplied, its allowance is included as in Corollary .

This implication does not construct a gravitational preparation, unitary evolution, access factorization, receiver, convergence theorem, or complement allowance.

Proof. Apply the finite theorem to the objects actually supplied by that model. The theorem proves an implication from those objects; it contains no construction of the gravitational hypotheses.

Endpoint topology and finite autonomous evolution

Lemma. For a fixed finite-dimensional code C, pointwise norm convergence of $V_t|\psi\rangle$ to $V_{*}|\psi\rangle$ for every $|\psi\rangle \in C$ implies operator-norm convergence.

Proof. For an orthonormal basis $\{e_i\}_{i=1}^{d}$ and $D_t = V_t - V_{*}$,

$$\|D_t\|_\infty \le \left(\sum_{i=1}^{d} \|D_t e_i\|^2 \right)^{1/2}.$$

Each summand tends to zero.

A spectrum limit or mass limit is not the pointwise isometry convergence in this lemma.

Proposition. Let H be a finite-dimensional time-independent Hamiltonian, $P_0: C \to H_{tot}$ an isometry, and

$$V_t = e^{-itH/\hbar} P_0$$

in a fixed output identification. If V_t has an operator-norm limit V_{*} as $t \to \infty$, then $V_t = P_0 = V_{*}$ for every t, and $HP_0 = 0$. This proposition does not cover a finite terminal time, a time-dependent dynamics, or a separately justified time-dependent output comparison.

Proof. For every $s \ge 0$,

$$e^{-isH/\hbar} V_{*} = \lim_{t \to \infty} V_{t+s} = V_{*}.$$

Therefore

$$\|V_t - V_{*}\|_\infty = \left\| e^{-itH/\hbar}(P_0 - V_{*}) \right\|_\infty = \|P_0 - V_{*}\|_\infty.$$

The left side tends to zero, so $P_0 = V_{*}$. The invariance under every s then gives $HP_0 = 0$.

Finite autonomous proxies therefore cannot be treated as irreversibly convergent evaporation models merely because they have many output registers. A common-output endpoint construction or a controlled limiting model remains a substantive obligation.

Finite insufficiency controls

Proposition. There is a complete finite energy-conserving process that:

- preserves the entire logical code globally;
- transfers all of a declared core-energy excitation to an outgoing factor;
- ends with zero core energy and zero accessible entropy;
- terminates after one unitary;
- has optimal outgoing recovery error q_d.

Proof. Let F and G be qubits and let $L \simeq C$ be a degenerate logical memory. Give the complete system Hamiltonian

$$H_{\mathrm{tot}} = \varepsilon(|1\rangle\langle 1|_F + |1\rangle\langle 1|_G) \otimes I_L, \qquad \varepsilon > 0.$$

Prepare

$$P_0|\psi\rangle = |1\rangle_F|0\rangle_G|\psi\rangle_L,$$

and apply the swap of F and G. It commutes with H_{tot}. Declare the outgoing access $R = G$ and complete residual $B = F \otimes L$. The resulting isometry is

$$V|\psi\rangle = |1\rangle_R|0\rangle_F|\psi\rangle_L.$$

The process is globally isometric, all core excitation has moved from F to G, and the core energy is zero. The accessible state is the fixed pure state $|1\rangle\langle 1|$, so its entropy is zero and its channel is constant. The program has one unitary step. Lemma gives outgoing recovery error q_d. The unknown logical code remains in the disclosed

residual memory L.

The core-energy quantity in this example is not identified with a black-hole mass. The example disproves the finite inference from an energy-drain ledger to a recovery endpoint.

Corollary. Suppose the *complete* residual has a positive Hamiltonian H_B with a one-dimensional zero-energy eigenspace spanned by $|b_0\rangle$. If

$$\mathrm{Tr}\,(H_B \mathscr{E}(\rho)) = 0 \quad \text{for every} \rho \in \mathrm{D}(C),$$

then $\mathscr{E} = \mathscr{T}_{|b_0\rangle\langle b_0|}$ and outgoing recovery is exact. Zero energy of a selected subsystem, or zero energy in a degenerate complete ground space, does not imply this conclusion.

Proof. A positive state of zero expectation for a positive Hamiltonian is supported on its kernel. The one-dimensional kernel therefore forces $\mathscr{E}(\rho) = |b_0\rangle\langle b_0|$ for every logical state. States linearly span the logical operator space, so the channel is the stated replacement channel. Theorem gives recovery. Proposition has a degenerate residual ground space containing L and proves the nonimplication.

Remark. The decoder optimization in is unrestricted within deterministic finite CPTP maps. It does not optimize live laboratory memory, streaming schedules, gate counts, receiver controls, or reusable apparatus. A smaller executable class can have strictly larger recovery error. The supplied En02-B and En02-C give model-specific streaming separations [CheungEn02B,CheungEn02C]. Their memory bounds are not imported as universal decoder bounds for the present interface.

The corrected Hawking-to-interface comparison

toprule Semiclassical datum Finite-interface question Permissible conclusion

midrule Thermal occupation factor and absorption probabilities Does a specified complete accessible channel satisfy a replacement-channel allowance? Thermal data alone do not prove it. If it is proved, Proposition gives a lower bound, and Corollary constrains complement allowances.

Mass decrease Is there a complete microscopic prepared dynamics with a justified energy ledger? An energy ledger is separate from logical recovery. Proposition supplies a finite counterexample to their identification.

Final disappearance discussed beyond the semiclassical regime Is there a same-model, access-preserving endpoint map? Definition states the sufficient obligations; Theorem is conditional on their construction.

Generalized entropy involving horizon area Does selected entropy determine code recovery? No: Proposition separates them. The dimension bound in Corollary is not an area law.

Particle creation from the incoming vacuum What is the complete preparation and its purification ledger? A vacuum emission calculation is not a microscopic logical-channel calculation. Selected residual constancy is insufficient, as Example proves.

Semiclassical metric and quantum matter Has complete gravitational unitary evolution been constructed? A classical background is an input to the semiclassical comparison. No complete quantum-gravitational dynamics is inferred here.

A classical radiation record Is the record being substituted for a quantum return? Record-only processing has the obstruction of Theorem . A retained quantum output requires its own access declaration.

bottomrule

The comparison preserves Hawking's calculation as an external semiclassical result. It neither reproduces the Bogoliubov coefficients nor turns a horizon mode decomposition into a physical tensor-factor and receiver theorem.

Documentary connections and inherited non-transfer

Every row below records a directed documentary edge from En07-C. Its attributes are

$$\texttt{role} = \textit{separation}, \qquad \texttt{declaredPath} = \textit{null}.$$

These are routing declarations, not theorem premises, model-identification maps, or compilation certificates.

The Cheung citations refer to the supplied versioned manuscripts. They do not assert independent verification of

the full corpus, and no exhaustive scan of that corpus is claimed.

toprule Documentary target Separation retained

midrule endfirsthead toprule Documentary target Separation retained

midrule endhead bottomrule endfoot

En07-A, thm:exact, and En07-B [CheungEn07A,CheungEn07B] Complete restoration, declared-access recovery, destructive discard, thermodynamic implementation, and approximate-bound direction remain different statements.

Hawk04-B, cor:gravity [CheungHawk04B] A conditional gravitational endpoint implication is not a constructed gravitational preparation, endpoint, or receiver.

Hawk04-A [CheungHawk04A] Complete-complement recovery is retained; an accessible replacement comparison is used only in the lower-bound direction proved here.

Hawk01-B [CheungHawk01B] A conserving sequential model and its entropy history are not a microscopic derivation of realistic evaporation.

Hawk03-A [CheungHawk03A] A finite partner interface and its algebraic recovery certificate are not a horizon dictionary or firewall resolution.

Hawk02-E [CheungHawk02E] A regulated interacting microscopic interface and finite-time approximation do not by themselves supply a small dynamical complete-residual bound or a gravitational endpoint.

En06-B [CheungEn06B] Exact preservation of a specified finite-network instrument is not a gravitational access construction. Its prescribed c is an input, not a derived invariant speed or metric.

Em03-E and Em03-H [CheungEm03E,CheungEm03H] Free-energy conservation, occupation calibration, and fixed-bath complementary certificates do not identify an evaporation endpoint or thermodynamic reset optimum.

En02-B and En02-C [CheungEn02B,CheungEn02C] Information-theoretic availability is distinguished from executable streaming recovery in their specified observer classes.

En04-A [CheungEn04A] Non-Clifford resources are not a complete-residual response certificate, and residual response is not geometric backreaction.

XIV-I, thm:I3, and XIV-J, thm:J2, cor:noadmission [CheungXIVI,CheungXIVJ] Constructed internal amplitudes and an off-characteristic supported reference are not the inherited complete characteristic map. No coefficient admission is transferred to En07-C.

XV-B, version 2, thm:lifting [CheungXVB] A relative-lifting interface is not a graph-specific constructed lift. Its supplied full-target verdict remains unresolved. The admission-dependent quantum exit is not opened.

XVI-N, thm:massless-obstruction [CheungXVIN] A conditional obstruction for the declared special instance is not a universal nonexistence result or a discharge of the separate prepared-source obligations.

XVI-O, rem:three-verdicts [CheungXVIO] Selected renormalization, bulk closure, and inherited-insertion status remain separate. O.1 stays NOT_CLOSED.

OpenAI catalogue documentary sheet [CatalogueSheet] Catalogue entries and family labels are documentary only: not mathematical premises, not verification of this series, not a compilation theorem, and not an Alexandrov–Zeeman identification.

The separate characteristic-map stop

For the supplied analytic versions, XIV-I constructs its named internal amplitude at I2 but leaves the complete joint characteristic operation unresolved at I3 [CheungXIVI]. XIV-J supplies reconstruction-descent conditions and a supported-sector reference away from the characteristic locus, but leaves its full required map unresolved at J2 [CheungXIVJ]. The supplied technical XV-B revision proves lifting and extension interfaces while recording its inherited full-target stop at B1 [CheungXVB].

Those are version-specific documentary statements, not a claim that every possible construction is impossible. En07-C supplies none of the missing analytic maps. In particular, its finite channel $\mathscr{E}$ is not the distribution-valued characteristic operator map of that programme.

The following separate analytic sequence is therefore not a construction performed in this manuscript:

$$\begin{gathered}\text{complete characteristic map}\\ \to \text{ joint section and controlled remainder}\\ \to \text{ uniform transverse estimates}\\ \to \text{ contact compatibility}\\ \to \text{ output wavefront and normalization}\\ \to Q_H R_1^{(1)} \in \mathcal{E}_H.\end{gathered}$$

No arrow is replaced by a finite decoder, a readout record, a dimension count, or a citation.

par **Standing routing refusals** par This is not spacetime emergence, not holographic reconstruction, and not a Page curve. The gravitational endpoint package is not constructed. The inherited full characteristic map is not constructed here. No interacting gravitational quantum coefficient is admitted. No compilation is asserted. O.1 stays NOT_CLOSED. XV-B's admission-dependent quantum exit is not opened. XVI-N's separate undischarged obligations are not discharged. These are scoped non-transfer statements, not universal nonexistence theorems.

par

Mathematical release and the same-model exit contract

Theorem. For the finite interface of Definition , this manuscript establishes:

• complete restoration and reference-stable trace-norm preservation;

• exact outgoing recovery if and only if the complete complement is constant, with the explicit decoder and dimension bound ;

• destructive basis Enter followed by forgetting equals partial trace, with a complete isometric dilation and no annihilation or thermodynamic-cost conclusion;

• record-only and featureless-accessible-channel lower bounds;

• approximate recovery/leakage conversion conditional on the explicitly imported KSW theorem, with the local half-diamond normalization and finite Choi and distinguishability certificates;

• explicit finite separations of selected thermal statistics, spectra, entropy, purifier marginals, energy drain, and termination from outgoing recovery;

• separation of complete energy, implementation-dependent erasure cost, and residual leakage;

• access-preserving endpoint, finite-time, and receiver error budgets conditional on the four same-model obligations;

• compatibility inequalities for simultaneous accessible replacement and complement-decoupling allowances. No gravitational preparation, complete gravitational evolution, outgoing gravitational receiver, bulk–boundary dictionary, Planck-scale endpoint, or inherited interacting gravitational quantum coefficient is constructed.

Proof. Theorem proves (i). Theorem and Corollary prove (ii). Lemma and Proposition prove (iii). Theorem and Proposition prove (iv). Convention , Theorem , Lemma , Proposition , and Lemma give (v). Examples and , Proposition , and Proposition prove (vi). Definition , Proposition , and Propositions and establish (vii). Theorem and Corollary prove (viii). Corollaries and prove (ix).

The excluded gravitational and analytic objects are not among these constructions. The documentary edges do not supply them.

Definition. For an application to the declared finite code, a quantum-return certificate must contain:

• a complete preparation and evolution preserving the code and its external references;

• justified outgoing quantum access and a deterministic decoder, with zero error or an explicit referenced allowance;

• an explicit classical-record ledger, not substitution of that record for the quantum return;

• an operational discard channel and, for any additional thermodynamic-cost claim, the implementation package of Definition ;

• an access-preserving endpoint comparison whenever an endpoint claim is made;

• no unsupported transfer into separately unresolved physical-model or analytic-admission obligations.

No thermal machine is required when no thermal-cost claim is made. No continuum endpoint is required when the claim is restricted to a proved finite-time interface. The certificate must match the actual scope of the claim.

Conclusion

The corrected finite-interface statement is

boxed

$$\begin{gathered}\text{complete isometry} \;\Rightarrow\; \text{joint restoration},\\ \text{constant complete complement} \;\Leftrightarrow\; ?\text{exact outgoing quantum recovery},\\ d_{\diamond}(\mathscr{E},\mathscr{T}_o)\le\zeta \;\Rightarrow\; ?\varepsilon_{\mathrm{rec}}\le \min\{q_d,\sqrt{2\zeta}\},\\ d_{\diamond}(\mathscr{N},\mathscr{T}_\omega)\le\eta \;\Rightarrow\; ?\varepsilon_{\mathrm{rec}}\ge \max\{0,q_d-\eta\},\\ \mathrm{Drop}_Z\mathrm{Enter}_B=\mathrm{Tr}_B,\\ \text{energy}\neq\text{erasure cost}\neq\text{residual leakage},\\ \text{gravitational endpoint implication: conditional}.\end{gathered}$$

Hawking's thermal emission result remains a semiclassical result. The finite interface supplies precise questions and certificates for outgoing quantum recovery. It does not answer those questions for gravity by changing the meaning of radiation, residual, observation, or endpoint.

Declarations

Mathematical dependency. Finite matrix and channel facts are used throughout. Stinespring dilation is a standard external structure theorem. The approximate information–disturbance estimate is explicitly imported from [KSW]; its normalization conversion is proved locally. No Cheung companion is required as a premise for the local finite proofs.

Documentary provenance. The unpublished Cheung entries below identify supplied manuscript IDs, versions, and selected labels. No publication venue, DOI, date, or fuller title is invented for those entries. Their complete proofs and corpus-wide relationships have not been independently audited here. The catalogue sheet is cited only as supplied documentary material.

Physical scope. No microscopic gravitational model, collapse-channel construction, coherent asymptotic receiver, thermal implementation optimum, Page curve, firewall resolution, holographic dictionary, or spacetime-emergence theorem is released.

Inherited analytic scope. No missing characteristic product is defined. No *R_1^{(1)}* is admitted, and no Ward differential is applied to an absent coefficient. The admission-dependent XV-B exit is not opened. XVI-N is not discharged. O.1 remains NOT_CLOSED.

Verification and submission. The source is written as one standalone pdfLaTeX file with an internal bibliography and no external figures or data files. No successful pdfLaTeX execution, proof-assistant verification, numerical execution, independent referee review, or journal acceptance is asserted. Before submission, the author should confirm metadata, archive the cited unpublished versions, run the build to convergence, check all references and equation links, and obtain independent checking of the mathematical arguments. These administrative and verification requirements do not replace any missing physical theorem.

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N. Cheung, *En07-A*, version 1. Supplied unpublished manuscript En07-A.tex. Selected labels: thm:global, thm:exact, lem:erasure-interface, prop:featureless, prop:no-landauer-from-channel, thm:release. Documentary comparison, not a premise.

N. Cheung, *En07-B*, version 1. Supplied unpublished manuscript En07-B.tex. Selected labels: def:interface, def:enter, thm:exact, thm:approx, prop:featureless, prop:no-landauer-from-channel, def:four, thm:endpoint, def:resolution, thm:release. Documentary comparison, not a premise.

N. Cheung, *Hawk04-A*, version 1. Supplied unpublished manuscript Hawk04-A.tex. Selected labels: thm:global, thm:exact, cor:nohiding, eq:normal-form, eq:exact-decoder. Documentary comparison, not a premise.

N. Cheung, *Hawk04-B*, version 2. Supplied unpublished manuscript Hawk04-B.tex. Selected labels: thm:classical, cor:gravity. Documentary comparison for destructive readout, termination, access, and the conditional gravitational endpoint.

N. Cheung, *En03-A*, version 1. Supplied unpublished manuscript En03-A.tex. Selected labels: def:enter, def:recording, def:nondestructive. Documentary comparison of three distinct measurement interfaces.

N. Cheung, *En04-A*, version 1. Supplied unpublished manuscript En04-A.tex. Selected labels: thm:global, thm:exact, lem:witness, cor:noimplication. Documentary comparison of complete preservation, residual response, and resource nonimplications.

N. Cheung, *Hawk01-B*, version 1. Supplied unpublished manuscript Hawk01-B.tex. Selected labels: thm:approximate, thm:entropyseparation, thm:release. Documentary comparison of a conserving sequential model, complete residual decoupling, and entropy-history separation.

N. Cheung, *Hawk03-A*, version 1. Supplied unpublished manuscript Hawk03-A.tex. Selected labels: thm:exact, thm:approx, thm:partner. Documentary comparison of a finite partner interface.

N. Cheung, *Hawk02-E*, version 1. Supplied unpublished manuscript Hawk02-E.tex. Selected labels: thm:regulator, thm:certificate, thm:endpoint-continuity, thm:release. Documentary comparison of a regulated interacting model; no gravitational endpoint is transferred.

N. Cheung, *En02-B*, version 1. Supplied unpublished manuscript En02-B-v1.tex. Selected labels: thm:B5, thm:tree. Documentary comparison of a specified streaming decoder's resource requirements.

N. Cheung, *En02-C*, version 1. Supplied unpublished manuscript En02-C-v1.tex. Selected labels: thm:C2, thm:C3, cor:robustgap. Documentary comparison of information-theoretic recovery and one-live-qubit streaming recovery.

N. Cheung, *En02-E*, version 2. Supplied unpublished manuscript En02-E-v2.tex. Selected labels: thm:gaussian, thm:cutoff. Documentary finite-packet and cutoff comparison.

N. Cheung, *En06-B*, version 1. Supplied unpublished manuscript En06-B.tex. Selected labels: thm:identity, thm:T, thm:release. Documentary finite-network transport comparison; the specified constant c is an input.

N. Cheung, *Em03-D*, version 1. Supplied unpublished manuscript Em03-D-v1.tex. Selected labels: thm:induced,

thm:packetbridge, thm:release. Documentary local bounded-observable comparison; not a coherent state-transfer theorem.

N. Cheung, *Em03-E*, version 1. Supplied unpublished manuscript Em03-E-v1.tex. Selected labels: thm:evolution, thm:channel, thm:matching, thm:release. Documentary finite dynamical and occupation-calibration comparison.

N. Cheung, *Em03-F*, version 1. Supplied unpublished manuscript Em03-F-v1.tex. Selected labels: thm:fieldchannel, thm:EB, thm:release. Documentary effective exterior-source comparison; classical source dependence is not arbitrary quantum recovery.

N. Cheung, *Em03-G*, version 1. Supplied unpublished manuscript Em03-G-v1.tex. Selected labels: thm:matchedrecovery, thm:stability, thm:release. Documentary ordered exterior-source comparison; a coherent local receiver is not transferred.

N. Cheung, *Em03-H*, version 8. Supplied unpublished manuscript Em03-H-v8.tex. Selected labels: lem:choi, thm:complementary. Documentary fixed-bath complementary certificate comparison.

N. Cheung, *Landmark XIV-I: Internal amplitudes and the joint-map stop*, version 1. Supplied unpublished manuscript XIV-I.tex. The descriptive subtitle identifies the supplied contents, not independently verified publication metadata. Selected labels: thm:I2, thm:I3, thm:release. Documentary non-transfer anchor.

N. Cheung, *Landmark XIV-J*, version 1. Supplied unpublished manuscript XIV-J.tex. Selected labels: prop:descent, lem:null, thm:reference, thm:J2, cor:noadmission, thm:release. Documentary non-transfer anchor.

N. Cheung, *Landmark XV-B*, version 2. Supplied unpublished manuscript XV-B.tex. Selected labels: thm:lifting, thm:jetnull, thm:pointcorrection, thm:release. Documentary lifting-interface comparison; the inherited full characteristic map remains unconstructed.

N. Cheung, *Landmark XVI-N*, version r12. Supplied unpublished manuscript XVI-N.tex. Selected label: thm:massless-obstruction. Documentary special-instance obstruction and scope anchor.

N. Cheung, *Landmark XVI-O*, version o70-sync. Supplied unpublished manuscript XVI-O.tex. Selected label: rem:three-verdicts. Documentary separation of construction, renormalized compatibility, and inherited-insertion status.

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