I-C-v2: The Shredder Interface

Destructive Readout, Black-Hole Analogues, and Quantum Computers Recovery obstructions, echo-sensitive records, and separate thermodynamic accounting on a prescribed conical product

Ngo Cheung

October 2026

Abstract

We develop the ``shredder'' analogy as a precise distinction between destructive classical readout and coherent quantum return. The underlying Enter interface consumes measured quantum names, retains every declared residual and failure, and returns finite classical words or stochastic symbols. Its principal quantitative consequence is uniform in the length and precision of any finite classical transcript: if the complete accessible output of an unknown d-dimensional code is classical, every deterministic decoder has reference-preserving half-diamond error at least $1 - 1/d$. The constant is sharp, and a complete implementation allowance δ weakens it by at most δ. The distinct Haar-average state-estimation infidelity bound is $(d - 1)/(d + 1)$. On a prescribed conical product, an explicitly declared scalar Friedrichs kernel supplies image arrivals and an apex-diffracted echo functional. Image-isolated windows distinguish prescribed deficits, and sufficiently short nonzero windows are injective on compact non-shadow sectors. These informative classical laws do not constitute quantum-return decoders. For neutral smooth cores, strong-resolvent convergence gives finite-precision indistinguishability of fixed smeared reports, not non-injectivity of finitely many exact real functionals. The horizon and quantum-computer comparisons are developed as analogies that the recovery obstruction, echo functional, and conditional thermal balance make quantitative. Append-only transcript entropy is monotone, whereas quantum radiation entropy is a different object. A finite thermal-reservoir identity replaces an unconditional positive-heat claim; readout alone determines neither heating nor an explosion. Explicit terminal continuations establish endpoint underdetermination. The contribution is an integrated, error-controlled account of how records can reveal geometry without restoring an arbitrary unknown quantum code. Established entanglement-breaking and thermodynamic principles are attributed rather than claimed as new. The analogy does not identify the cone apex with a horizon or Hawking radiation with a classical register. No Page curve, evaporation dynamics, physical spin-2 Hessian, Einstein source law, or spacetime emergence is constructed.

Bibliographic record

Title	I-C-v2: The Shredder Interface
Author	Cheung, Ngo
Publication date	2026/10/01
Institution	Enter
Report number	05-I-C-v2
Publisher	Cheung Ngo Medical Limited
First page	1
Last page	29
Language	en

Introduction: Enter, access, and the corrected question

Names, outputs, and philosophical orientation

In Cheung's Enter calculus, runtime quantum names are access rights to tensor factors of a quantum state. Destructive computational-basis readout consumes the measured names and supplies classical outcomes. The concrete Born semantics and its characterization within a specified strategy class are distinct layers of that

development [CheungEnter,CheungI]. Here the Born channel formalism is assumed. We do not rederive that characterization from an abbreviated list of requirements.

The introductory essay *The Philosophy of Enter* emphasizes four separations: destructive observation is not coherent recovery; a classical record is not a quantum-return decoder; a channel is not a thermal machine; and an analogue is not a physical identification [CheungPhil]. Its role here is documentary orientation, not a theorem premise. The related En10-A interface discussion also distinguishes parameter inference, access rights, complete resources, and physical correspondence [CheungEn10].

The spacetime model I-B installs finite names, locations, accessible algebras, complete controls and resources, and calibrated finite clocks on an already prescribed continuum geometry [CheungIB]. Consuming a clock encoding does not consume the geometric time axis. A clock-basis change is not generally a spacetime coordinate change.

The mathematical question

The useful operational content of the shredder picture is

$$\text{boxed}\begin{array}{l}\text{unknown quantum code} \;\to\; \text{complete classical-only accessible output}\\ \qquad\Rightarrow \text{no universal reference-preserving quantum return.}\end{array}$$

The word *complete* rules out an omitted receiver memory, quantum reservoir, entangled resource, or other available message-bearing factor.

In the vernacular, this is the shredder: ``quantum name in, classical record plus heat out.'' The precise reading is

$$\text{quantum name in} \;\to\; \text{classical report at the declared access boundary,}$$

together with a *separately specified* physical implementation ledger. Heat production belongs to that ledger when its hypotheses establish it; it is not an automatic term in a channel codomain. The theorems below make the access, error metric, and thermodynamic claims precise.

The recovery statement is not a global information-destruction theorem. Name consumption, tracing out a factor, recording an outcome, resetting a memory, and heating a reservoir are different operations. Their distinctions are central, not qualifications added after the proof.

Repairs that the analogy must retain

• The bound $1 - 1/d$ concerns a referenced channel metric. An unspecified ``average or worst-case error'' has no such universal interpretation.

• A finite outcome alphabet can have an injective probability law on a continuum of parameters. Exact laws, realized finite records, and quantum-return channels are different objects.

• Append-only Shannon entropy is not the fine-grained von Neumann entropy of unmeasured radiation.

• Classical heat readings do not grant coherent quantum access to the reservoir that produced them.

• Consuming one name need not consume information already transferred to another live quantum register.

• Readout does not specify a Hamiltonian, heat capacity, cooling law, mass budget, stopping time, or endpoint.

• A nonempty returned quantum residual prevents the complete output from being called classical-only merely because some other outputs are classical.

• A last-moment quantum-output channel changes the exclusive classical-access hypothesis; it does not evade its consequences while retaining that hypothesis.

Provenance and logical independence

Connections to other Enter sheets are provenance comparisons or separations, not automatic theorem transfers and not a declared path through the programme. All finite-channel arguments needed here are proved locally.

The continuum application uses an explicitly stated kernel input. The restricted neutral-core proof is also included. The supplied full I-B-ECHO-v2.tex is cited version-specifically [CheungEcho]. Other supplied excerpts and summary-level descriptions are identified as such. A theorem label in an unavailable source is not treated as its proof.

The standard channel, instrument, state-estimation, and thermodynamic principles used here belong to the

established literature [DaviesLewis,HSR,Watrous,MassarPopescu,NielsenAverage,ReebWolf]. The contribution is their corrected integration with the prescribed-host echo interface and the explicit limits of the analogy.

The vernacular analogy and its mathematical dictionary

Originals, names, and reports

Imagine an original photographic negative being passed to a machine that returns a report but not the original. A quantum name plays the role of the original only in the limited sense that it names an available quantum input. The unknown state may contain coherence and correlations with an untouched reference that the report does not return.

The metaphor should not be read as a claim that every quantum state is intrinsically short-lived or that all physical carriers vanish. Nor is a quantum state an ordinary classical image whose pixels merely await sufficiently careful inspection. The impossibility of universally cloning an unknown state already distinguishes these tasks [WoottersZurek]. The recovery obstruction proved here uses the stronger, task-specific condition of classical-only access, not the no-cloning theorem alone.

The machine's ``strips'' are classical codes or stochastic symbols. In Proposition , these are outcomes in a finite diagonal register. In the echo application, the continuum window functional first requires a supplied finite preparation; only then does the complete instrument produce its finite symbols.

A strip may be useful. It can witness an input-dependent probability, support statistical inference under a declared likelihood, or distinguish two prescribed deficits. Usefulness does not mean universal restoration of the original unknown quantum code. Theorems and quantify that distinction.

Three outputs that must not be collapsed

The informal machine has three separately specified accounts:

• a public classical report R;

• a quantum residual A, if any, with declared ownership;

• a resource and energy ledger for the implementing apparatus. If A remains accessible, the interface is generally classical–quantum. If A is inaccessible, the decoder is not permitted to use it merely because the metaphor imagines that ``the machine must still have something inside.''

Heat is an energy transfer under a physical model. A classical heat-meter label can be appended to R. The microscopic reservoir is not thereby converted into a classical factor, and its quantum correlations have not been accounted for by one energy number. This is the distinction behind Theorem and Definition .

toprule Vernacular term Mathematical object Limit of the translation

midrule Original Unknown state on a named finite input Q. Not an ontology of all implementing matter.

Shredder mouth Specified destructive instrument and access cut. Not automatically a horizon, apex, or physical detector.

Shredded report Finite classical history, including failures. Not an exact probability law or universal quantum return.

Stuff still inside Declared quantum residual and complete inaccessible output. Not omitted from the ledger; not necessarily a copy.

Heat Reservoir energy change in a named implementation. Not fixed by the abstract channel or name count.

Running out A stopping rule or finite no-allocation resource condition. Not evaporation or exhaustion of physical energy.

bottomrule

What ``gone'' means

After destructive Enter, the measured name is gone from the returned quantum context. A continuation cannot request that same input name again. This is the precise sense of ``gone'' used in the manuscript.

It is stronger than merely hiding a record from a display, but weaker than asserting fundamental annihilation of quantum information. A complete unitary implementation may preserve the input in its full joint output while its

public classical restriction does not support a decoder. Proposition supplies the elementary restoration statement, and Example shows why consumed names and transferred logical states cannot be identified.

Prescribed spacetime and complete finite instruments

Geometry, clocks, and causal certificates

Fix

$$0 < \alpha \le 1, \qquad \mu = 2\pi(1-\alpha),$$

and prescribe, in units $c = 1$,

$$\begin{aligned} M &= \mathbb{R}_t \times (0,\infty)_r \times (\mathbb{R}/2\pi\mathbb{Z})_\varphi \times \mathbb{R}_z, \\ g &= -\mathrm{d}t^2 + \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2. \end{aligned}$$

The completion adds the worldsheet

$$\mathscr{W} = \mathbb{R}_t \times \{o\} \times \mathbb{R}_z,$$

not a worldline. The spatial cone and the Lorentzian causal cones are different uses of the word ``cone."

A flat complex line bundle, when used, has prescribed angular holonomy $e^{2\pi i\beta}$, $\beta \in [0, 1)$. The spatial self-adjoint extension is also prescribed. None of these data is selected by readout [CheungIB,CheungIBG].

A finite clock has a chosen orthonormal basis $\{|\tau_j\rangle\}$ and a supplied assignment of finite outcome words to geometric labels or scheduled events. This is calibration data, not a determination of coordinate time from an arbitrary clock observable [CheungIB,CheungEn09].

When a circuit is given a spacetime interpretation, every communication or classical-control dependency requires a causal certificate. Transitivity then contains the scheduled ancestors in the appropriate metric past. A finite instruction tree alone does not construct that embedding, and its reachability relation is not automatically the whole metric causal past [CheungVIIJ,CheungXK].

Notation and access contract

For a finite-dimensional space Q, let $\mathscr{B}(Q)$ be its matrix algebra and $\mathscr{D}(Q)$ its density operators. Channels are completely positive and trace preserving (CPTP). We use

$$d_{\mathrm{tr}}(\rho,\sigma) = \frac{1}{2}\|\rho-\sigma\|_1, \qquad d_\diamond(\mathscr{N},\mathscr{M}) = \frac{1}{2}\|\mathscr{N}-\mathscr{M}\|_\diamond.$$

The second metric includes untouched references [Watrous]. Entropies are in bits except where natural logarithms are explicitly used in thermodynamics.

A finite classical register R has the observable algebra

$$\mathscr{A}_R = \Big\{ \sum_x a_x |x\rangle\langle x| : a_x \in \mathbb{C} \Big\}.$$

Representing it by matrices does not grant access to coherent superpositions of its recorded labels.

A deterministic quantum-return decoder acts only on the declared accessible output. It may use fresh input-independent ancillas and randomness. It has no access to the untouched reference or another message-bearing quantum factor. In particular, no pre-shared entanglement crossing the cut is supplied unless it is explicitly included in a different access contract.

The last restriction is essential. Teleportation can return an unknown state using a classical message *and* a previously shared entangled quantum resource; that is not decoding from a classical register alone [Teleportation].

The destructive instrument

Proposition. Let $Q \cong \mathbb{C}^d$, let A contain every returned quantum register, and choose an orthonormal basis $\{|j\rangle\}_{j=0}^{d-1}$ of Q. Put

$$D_j = \langle j|_Q \otimes I_A.$$

Then

$$\mathscr{E}^{A}_{Q\to R}(X)=\sum_{j=0}^{d-1}\operatorname{proj} j_R\otimes D_jXD_j^{\dagger}$$

is CPTP. For every untouched finite reference E,

$$\operatorname{Tr}_R[(\mathscr{E}^{A}_{Q\to R}\otimes \mathrm{id}_E)(X)]=\operatorname{Tr}_Q X.$$

The factor Q is absent from the returned quantum interface. For a state ρ, branch j has probability $Tr\ (D_j\rho D_j\hat{}\{\dagger\})$; its residual is normalized only on a positive-probability branch.

Proof. The recorded Kraus operators are $L_j = |j\rangle_R \otimes D_j$, and

$$\sum_j L_j^{\dagger}L_j=\sum_j \operatorname{proj} j_Q\otimes I_A=I_{QA}.$$

Thus the map is CPTP. Expand

$$X=\sum_{j,k}\operatorname{ketbra} jk_Q\otimes X_{jk}.$$

The recorded blocks are X_{jj}, and their sum is $Tr_Q X$. The same calculation holds when each block acts on $A\otimes E$.

This is the local complete identity associated with I-B, En09-A, X-J-ST, and I-B-ECHO [CheungIB,CheungEn09,CheungXJ,CheungEcho]. It formalizes the name-consumption part of Section , not a heat-production law.

More generally, a finite instrument is a family of CP maps $\{\mathscr{I}_x\}$ whose sum is trace preserving [DaviesLewis]. Codes, timeout symbols, assignment errors, and failures can all be retained in x. No failure is deleted from the attempted-run law.

Lemma. A finite instruction tree with exclusively classical accessible terminal output has channel

$$\mathscr{C}(\rho)=\sum_x \operatorname{Tr}(E_x\rho)\operatorname{proj} x,\qquad E_x\ge 0,\quad \sum_x E_x=I.$$

Adaptive controls, fixed auxiliary preparations, and hidden quantum residuals do not change this conclusion.

Proof. Compose the CP maps on each complete history and trace every inaccessible quantum output. Its probability is a positive linear functional of ρ, hence is $Tr\ (E_x\rho)$ for an effect $E_x \ge 0$. Completeness gives $\sum_x E_x = I$. A stopping branch is retained as a history and may be padded by fixed null symbols.

Lemma. For $\mathscr{C}$ in and any decoder $\mathscr{D}: R \to Q\hat{}\{'\}$, there are states σ_x such that

$$\mathscr{D}\mathscr{C}(\rho)=\sum_x \operatorname{Tr}(E_x\rho)\sigma_x.$$

For every referenced input, the decoded output is separable across reference versus $Q\hat{}\{'\}$.

Proof. Set $\sigma_x = \mathscr{D}(\operatorname{proj} x)$. For ρ_{EQ}, the output is

$$\sum_x \rho_E^x\otimes\sigma_x,$$

where

$$\rho_E^x=\operatorname{Tr}_Q\left[(I_E\otimes\sqrt{E_x})\rho_{EQ}(I_E\otimes\sqrt{E_x})\right]\ge 0.$$

Its nonzero blocks normalize to product states. Their traces sum to one.

This is the standard entanglement-breaking mechanism [HSR]. It is the precise replacement for the claim that a collection of classical strips can never be glued into a universal quantum-return channel.

Proposition. Suppose two n-stage processes have the same intermediate types and complete stage channels $\mathscr{F}_{\ell}, \mathscr{F}_{\ell}$, with

$$d_\diamond(\widehat{\mathscr{F}}_\ell,\mathscr{F}_\ell)\le\delta_\ell.$$

History registers, controls, failures, and surviving factors are included in these types. Then

$$d_\diamond(\widehat{\mathscr{F}}_n\cdots\widehat{\mathscr{F}}_1,\mathscr{F}_n\cdots\mathscr{F}_1)\le\min\left\{1,\sum_{\ell=1}^{n}\delta_\ell\right\}.$$

Proof. Subtract the compositions and telescope by replacing one stage at a time. Pre- and post-composition by CPTP maps cannot increase the diamond norm. Apply the triangle inequality and the maximal half-diamond distance one between channels.

This proves the finite error fact used below. It does not construct a physical realization or the resource-fit certificates distinguished in X-K-ST [CheungXK].

The recovery obstruction and its correct metrics

Let $Q^{'} \cong Q \cong \mathbb{C}^d$, with a fixed intended identification. Define

$$|\Phi_d\rangle = \frac{1}{\sqrt{d}} \sum_{j=0}^{d-1} |j\rangle_E |j\rangle_Q, \qquad \Phi_d = |\Phi_d\rangle\langle\Phi_d|.$$

For a channel $\mathscr{T}: Q \to Q^{'}$, put

$$F_e(\mathscr{T}) = \mathrm{Tr}\,[\Phi_d (\mathrm{id}_E \otimes \mathscr{T})(\Phi_d)].$$

This is its maximally mixed-input entanglement fidelity.

Let $d \geq 2$. For every finite classical-output channel $\mathscr{C}: Q \to R$ and deterministic decoder $\mathscr{D}$,

$$F_e(\mathscr{D}\mathscr{C}) \leq \frac{1}{d}, \qquad d_\diamond(\mathscr{D}\mathscr{C}, \mathrm{id}_Q) \geq 1 - \frac{1}{d}.$$

Both constants are sharp over this class.

For the distinct Haar-average pure-state task

$$F_{\mathrm{av}}(\mathscr{T}) = \int \langle\psi|\mathscr{T}(\mathrm{proj}\,\psi)|\psi\rangle \, \mathrm{d}\psi,$$

one instead has

$$F_{\mathrm{av}}(\mathscr{D}\mathscr{C}) \leq \frac{2}{d+1}.$$

Thus its average infidelity is at least $(d - 1)/(d + 1)$, not $1 - 1/d$.

Proof. For states a, b on $\mathbb{C}^d$,

$$\mathrm{Tr}\,[\Phi_d (a \otimes b)] = \frac{1}{d}\mathrm{Tr}\,(a^{\mathrm{T}} b) \leq \frac{1}{d}.$$

Convexity extends this to every separable state. Lemma proves the fidelity bound. Testing the output difference with the effect Φ_d gives

$$d_{\mathrm{tr}}((\mathrm{id}_E \otimes \mathscr{D}\mathscr{C})(\Phi_d), \Phi_d) \geq 1 - F_e(\mathscr{D}\mathscr{C}) \geq 1 - \frac{1}{d}.$$

This lower-bounds the half-diamond distance.

For sharpness, measure and reprepare one fixed basis. The decoded channel is the pinching

$$\Delta(\rho) = \sum_j \mathrm{proj}\, j \rho \mathrm{proj}\, j.$$

For $Z = \sum_{j=0}^{d-1} e^{2\pi i j/d} \mathrm{proj}\, j$,

$$\Delta = \frac{1}{d} \sum_{k=0}^{d-1} \mathrm{Ad}_{Z^k}.$$

Consequently,

$$d_\diamond(\Delta, \mathrm{id}) \leq \frac{1}{d} \sum_{k=1}^{d-1} d_\diamond(\mathrm{Ad}_{Z^k}, \mathrm{id}) \leq \frac{d-1}{d}.$$

The lower bound gives equality, and $F_e(\Delta) = 1/d$.

The Haar second-moment identity is

$$\int \mathrm{proj}\,\psi \otimes \mathrm{proj}\,\psi \, \mathrm{d}\psi = \frac{I + \mathrm{S}}{d(d+1)},$$

where S swaps the two factors. Unitary invariance, symmetric support, and normalization prove it. For $\mathscr{T}(\rho) = \sum_x$

Tr (E_xρ)σ_x, it yields

$$F_{\mathrm{av}}(\mathscr{T}) = \frac{\sum_x (\mathrm{Tr}\, E_x + \mathrm{Tr}\,(E_x\sigma_x))}{d(d+1)} = \frac{1 + dF_e(\mathscr{T})}{d+1}.$$

Apply the first bound. Pinching attains *2/(d + 1)*.

par *Interpretive boundary*. The statement concerns declared finite channels and, where used, free operators, kernels, and supplied preparations on prescribed continuum data. Destructive Enter does not acquire an unknown continuum functional, select the host geometry, or turn a conserved tensor into a source. No geometry update, Einstein dynamics, or spacetime emergence is inferred.

Theorem is the quantitative content of the ``original versus report'' distinction in Section . The underlying principles are established [HSR,Watrous,MassarPopescu,NielsenAverage]. The classical-diary comparisons in Cheung's Hawk records retain their own models [CheungHawkClassical]; no unprovided theorem is used in this proof.

Corollary. Suppose $\mathscr{N}: Q \to Z$ satisfies

$$d_\diamond(\widehat{\mathscr{N}}, \mathscr{C}) \le \delta$$

for a classical-output comparison channel embedded in the same output space *Z*. Then every decoder obeys

$$d_\diamond(\mathscr{D}\widehat{\mathscr{N}}, \mathrm{id}_Q) \ge [1 - \frac{1}{d} - \delta]_+.$$

The implemented output need not be exactly classical.

Proof. Use post-composition contractivity and

$$d_\diamond(\mathscr{D}\widehat{\mathscr{N}}, \mathrm{id})\mathrm{ged}_\diamond\,(\mathscr{D}\mathscr{C}, \mathrm{id}) - d_\diamond(\widehat{\mathscr{N}}, \mathscr{C}).$$

Apply Theorem .

Remark. Known orthogonal states can be measured and re-prepared perfectly. For a uniformly unknown qubit pure state, the average infidelity bound is *1/3*, whereas the referenced channel obstruction is *1/2*. These are not contradictory constants [MassarPopescu,NielsenAverage].

Multiple independently prepared copies are additional inputs. Tomography on them is not repeated access to the same consumed name. Heralded success likewise requires a separately specified trace-nonincreasing task; deleting failure trials does not preserve the deterministic channel in Theorem .

Proposition. For every *d ≥ 2*, a finite-outcome POVM can have a complete probability law determining any density matrix on ℂ^d. This does not provide a deterministic one-shot quantum-return decoder.

Proof. Choose a traceless Hermitian basis *G_1, …, G_m, m = d^2 − 1*, scaled so that *‖G_a‖_{∞} ≤ 1*. Set

$$E_{a,\pm} = \frac{I \pm G_a}{2m}.$$

These effects are positive and sum to *I*. Their probability differences determine *Tr (G_aρ)*, hence *ρ*. A complete exact law is not one realized finite outcome. Theorem still applies.

No effective code dimension is inferred from a short continuum time window alone. A dimension *d* requires a declared code, cutoff, or preparation subspace.

The horizon analogue and the information question

A mapping with an unconstructed physical bridge

In the horizon story, the interface is imagined as the place where input names cease to be available to an exterior observer. Near-horizon modes play the role of inputs; an exterior observer retains reports.

On the conical host, one can draw a suggestive picture in which the apex worldsheet occupies that position in the diagram. This is not a geometric identification. The apex worldsheet in is not an event horizon, and the model supplies no evaporating black-hole metric. The interface cut is a declaration of access, whereas a horizon is a causal-geometric object.

The scalar kernel below separates direct and image arrivals from an apex-diffracted wake. That wake is the continuum object that motivates calling an echo a ``report from the interface." Its windows can reveal a prescribed deficit.

The finite record is not the wake itself. A separate preparation assumption connects a requested window value to a finite register. Without a field–apparatus construction, the finite instrument has not acquired the wake. The Sommerfeld–Carslaw-type representation and finite propagation provide a free-wave calculation, not Hawking emission [CheegerTaylorI,CheegerTaylorII,CheungIBG,CheungEcho].

toprule Horizon narrative Finite or continuum counterpart Missing identification

midrule Infalling or near-horizon input Declared quantum names or an independently supplied preparation. Physical code and local loading dynamics.

Horizon/interface Access cut; diagrammatically, the cone apex. A horizon geometry and its relation to that cut.

Outgoing flux A free exterior response and its requested echo functional. A Hawking state, emission channel, and detector coupling.

Exterior observation Complete classical transcript, including failures and heat labels. Proof that this is the complete physical accessible algebra.

Evaporation endpoint A chosen terminal channel and residual ledger. Gravitational endpoint dynamics and backreaction.

bottomrule

What the recovery obstruction says

The question ``Can the exterior strips restore the original?" has a mathematical answer only after ``exterior" and ``restore" are specified. If exterior access is exclusively classical and restoration means the identity channel on an arbitrary unknown d-dimensional code, Theorems and give the error lower bound $1 - 1/d$.

This is not a resolution of the physical black-hole information problem. It proves a conditional obstruction for one proposed output interface. Whether a physical process has that interface, or instead retains accessible quantum correlations, is additional physics. A nonclassical return channel or extra quantum access changes the premise rather than contradicting the theorem.

Hawking radiation is not a classical register

Hawking's semiclassical calculation concerns quantum fields and outgoing observables [Hawking1975]. A thermal occupation law does not specify a complete classical-output channel. Nor does a thermal-looking marginal determine every correlation in a joint radiation state.

A detector can convert selected radiation observables into a classical log. That readout is another operation. Quantum recovery from radiation before readout and recovery from the log afterward are different tasks. Cheung's complete-output and exterior-source records preserve related distinctions within their stated models [CheungHawk04A,CheungEm03]. No Page curve, spectrum, or evaporation channel is derived by calling the conical echo ``Hawking-like."

Apex-diffracted reports and geometric sensitivity

An explicit scalar continuum input

Prescribe the nonnegative Friedrichs spatial operator $A_{F, \beta}$ and the wave convention

$$P = \partial_t^2 + A_{\mathrm{F},\beta}.$$

Its retarded propagator is

$$G_+(t) = \theta(t)\frac{\sin\left(t\sqrt{A_{\mathrm{F},\beta}}\right)}{\sqrt{A_{\mathrm{F},\beta}}},$$

with continuous sine-quotient value t at spectral value zero.

For a lifted angular difference $\theta = \alpha(\varphi - \varphi^{\prime})$, define

$$K(\theta) = \{k \in \mathbb{Z} : |\theta - 2\pi\alpha k| < \pi\},$$
$$R_k^2 = r^2 + r'^2 - 2rr'\cos(\theta - 2\pi\alpha k) + \Delta z^2,$$
$$\rho(u)^2 = r^2 + r'^2 + 2rr'\cosh u + \Delta z^2, \qquad \rho_0 = \rho(0).$$

At noncoincident regular spatial pairs off

$$\theta \pm \pi \in 2\pi\alpha\mathbb{Z},$$

use, relative to $dV_0 = \alpha r\, dr\, d\varphi\, dz$,

$$G_+(t; x, x') = \sum_{k \in K(\theta)} e^{2\pi i\beta k} \frac{\delta(t - R_k)}{4\pi R_k} - \theta(t - \rho_0)\frac{\Phi_\beta(u_0, \theta)}{4\pi^2 \alpha r r' \sinh u_0},$$

where

$$\cosh u_0 = \frac{t^2 - r^2 - r'^2 - \Delta z^2}{2rr'}.$$

For $u > 0$,

$$\Phi_\beta(u, \theta) = \sum_{n \in \mathbb{Z}} \sin\left(\frac{\pi|n + \beta|}{\alpha}\right) e^{-|n + \beta \mathrm{rvertu}/\alpha} e^{i(n + \beta)\theta/\alpha}.$$

In the neutral case,

$$\Phi_0(u, \theta) = \frac{1}{2}\sum_{\pm} \frac{\sin((\pi \pm \theta)/\alpha)}{\cosh(u/\alpha) - \cos((\pi \pm \theta)/\alpha)}.$$

At shadow values, the total joint distribution is intended, not separate substitution into singular geometric and diffracted pieces. Intrinsic finite propagation for the Friedrichs wave group is also retained.

These are explicit analytical inputs matching the scalar representation stated in I-B-G and the supplied I-B-ECHO-v2 [CheungIBG,CheungEcho]. I-C does not independently establish the entire singular Green-operator theory.

A general Green inverse depends on its extension and test domain. In particular, an extension-dependent time-compact spectral exact sequence is not automatically an exactness theorem on the usual compact-spacetime test space. The supplied I-B-G excerpts explicitly retain that distinction [CheungIBG].

Neutral backscatter and admissible windows

Set

$$\beta = 0, \quad \theta = 0, \quad \Delta z = 0, \quad 0 < r_e < r_r, \quad R_0 = r_r - r_e, \quad L = r_e + r_r.$$

Let $f \in C_c^{\infty}(\mathbb{R})$ be nonnegative, supported in $[0, T_s]$, with integral one. Temporal convolution of gives

$$u_{\mathrm{dif}}(t) = -\frac{1}{4\pi^2\alpha}\int_0^\infty \Phi_0(u, 0)\frac{f(t - \rho(u))}{\rho(u)}\,\mathrm{d}u.$$

Indeed, $\rho^{\prime}(u) = r_e r_r \sinh u/\rho(u)$ supplies the Jacobian. The diffracted contribution vanishes for $t < L$.

Exclude

$$\mathscr{S} = \{1/(2m) : m \in \mathbb{N}\},$$

and define

$$\mathscr{I}_0 = (1/2, 1], \qquad \mathscr{I}_m = (1/(2m + 2), 1/(2m)), \quad m \geq 1.$$

Fix a compact interval J in one sector. For $\alpha \in \mathscr{I}_m$,

$$K(0) = \{-m, \ldots, m\},$$

with distinct lengths

$$R_j(\alpha) = \sqrt{r_e^2 + r_r^2 - 2r_e r_r \cos(2\pi\alpha j)}, \quad j = 0, \ldots, m.$$

For $m \geq 1$, $R_0 < \cdots < R_m < L$. The defining angles lie in $[0, \pi)$, which proves the order.

Compactness gives positive margins between the direct and next arrival, and between the last arrival and L. Choose T_s smaller than the relevant margins. For $m = 0$, only the latter condition is required.

Choose a bounded nonnegative direct gate χ_d satisfying

$$\chi_d = 1 \quad \text{on} [R_0, R_0 + T_s],$$

with support excluding every image pulse and every diffracted signal for $\alpha \in J$. Merely intersecting or containing the direct pulse support without the normalization would not give the calibration below.

Choose a bounded nonnegative echo gate χ_e supported in $[L, L + T_w]$. All geometric pulses have ended before L. Put

$$y_d = \int \chi_d(t) u(t)\, \mathrm{d}t, \qquad y_e = \int \chi_e(t) u(t)\, \mathrm{d}t,$$
$$W(u) = \int \chi_e(t) f(t - \rho(u))\, \mathrm{d}t,$$
$$\cosh u_* = \frac{(L + T_w)^2 - r_e^2 - r_r^2}{2 r_e r_r},$$
$$N_W = \frac{R_0}{\pi} \int_0^{u_*} \frac{W(u)}{\rho(u)}\, \mathrm{d}u > 0, \qquad s(\alpha) = \frac{y_e / y_d}{N_W}.$$

The strict positivity excludes a zero gate or absent pulse overlap. The geometry, pulse, and gates are common to all $\alpha \in J$.

Proposition. Define

$$q(\alpha, u) = -\frac{\sin(\pi/\alpha)}{\alpha(\cosh(u/\alpha) - \cos(\pi/\alpha))},$$
$$q_0(\alpha) = -\frac{1}{\alpha} \cot \frac{\pi}{2\alpha}, \qquad D(\alpha) = 1 - \cos(\pi/\alpha).$$

Then $y_d = 1/(4\pi R_0)$, and

$$s(\alpha) = \frac{\int_0^{u_*} q(\alpha, u) W(u) \rho(u)^{-1}\, \mathrm{d}u}{\int_0^{u_*} W(u) \rho(u)^{-1}\, \mathrm{d}u}.$$

On every non-shadow sector,

$$q_0'(\alpha) \le -\pi/2,$$
$$|s(\alpha) - q_0(\alpha)| \le \varepsilon_W(\alpha, u_*) := \frac{|\sin(\pi/\alpha)|(\cosh(u_*/\alpha) - 1)}{\alpha D(\alpha)^2}.$$

If $E_J = \max_J \varepsilon_W(\alpha, u_\{*\})$, then

$$|s(\alpha_1) - s(\alpha_2)| \ge \frac{\pi}{2} |\alpha_1 - \alpha_2| - 2E_J.$$

On $1/2 < \alpha \le 1$, the functional vanishes exactly at $\alpha = 1$.

For sufficiently short nonzero gates preserving $N_W > 0$ and image isolation, s is strictly decreasing on J.

Proof. The direct normalization and pulse integral give y_d. Integrate against χ_e, then divide by y_dN_W. The relevant u-range is compact and the denominator is positive, proving .

For $H = \cosh(u/\alpha) - 1$,

$$q(\alpha, u) = q_0(\alpha) \frac{D(\alpha)}{D(\alpha) + H}.$$

This proves and $|s(\alpha)| \le |q_0(\alpha)|$. For $1/2 < \alpha < 1$, the integrand q is strictly positive; at $\alpha = 1$ it vanishes identically. Positive normalization proves the witness statement.

With $x = \pi/(2\alpha)$,

$$q_0'(\alpha) = -\frac{x - \sin x \cos x}{\alpha^2 \sin^2 x}.$$

For $x \ge \pi/2$, the numerator is at least $\pi/2$, because its derivative is $2\sin\hat{}2\, x$. The denominator is at most one. This

proves , and the mean value theorem plus proves .

An explicit finite-gate criterion is useful. Let

$$a = \min J, \quad d_* = \min_J D > 0, \quad M_0 = \max_J |q_0|, \quad M_1 = \max_J |q_0'|.$$

The estimates

$$H \le \frac{u^2}{2a^2}\cosh(u_*/a), \qquad |\partial_\alpha H| \le \frac{u^2}{a^3}\cosh(u_*/a), \qquad |D'| \le \frac{\pi}{a^2}$$

and differentiation of *q − q_0 = −q_0H/(D + H)* give

$$|\partial_\alpha(q - q_0)| \le L_J(u_*),$$

where

$$L_J(u_*) = u_*^2\cosh(u_*/a)\left[\frac{M_1}{2a^2 d_*} + \frac{M_0}{a^3 d_*} + \frac{\pi M_0}{2a^4 d_*^2}\right].$$

The averaging measure is independent of α. Hence

$$s'(\alpha) \le -\pi/2 + L_J(u_*).$$

If *L_J(u_{*}) < π/2*,

$$|s(\alpha_1) - s(\alpha_2)| \ge (\pi/2 - L_J(u_*))|\alpha_1 - \alpha_2|.$$

The criterion tends to zero with *u_{*}*. This asserts short-gate availability only for pulse/gate families retaining nonzero overlap and isolation.

par *Interpretive boundary*. The statement concerns declared finite channels and, where used, free operators, kernels, and supplied preparations on prescribed continuum data. Destructive Enter does not acquire an unknown continuum functional, select the host geometry, or turn a conserved tensor into a source. No geometry update, Einstein dynamics, or spacetime emergence is inferred.

The required scalar algebra is reproduced locally [CheungEcho]. There is no global fixed-resolution theorem through shadow transitions. At neutral pure-image openings *α = 1/N* with odd *N > 1*, the diffracted weight also vanishes. The flat-space witness is therefore restricted to its stated range.

The normalized echo can distinguish geometric parameters, sometimes injectively as an exact functional. In this sense the strip is informative rather than blank. But knowing a prescribed deficit is a classical parameter task. It is not the task of returning every unknown state of a quantum code with its reference entanglement. The zero-deficit witness does not reverse a destructive cut.

Supplied scalar values and finite report laws

For *σ > 0*, gain *γ ≠ 0*, and failure rate *0 ≤ r < 1*, define

$$p_\sigma(z) = \frac{1 + \tanh(z/\sigma)}{2},$$

$$|e_\sigma(z)\rangle = \sqrt{1 - p_\sigma(z)}|0\rangle + \sqrt{p_\sigma(z)}|1\rangle, \qquad \tau_\sigma(z) = |e_\sigma(z)\rangle\langle e_\sigma(z)|.$$

Assumption. For a requested task value *s(α)*, a preparation interface supplies a finite code or enclosure, with failures and range conventions, and an approximation *s̃(α)* satisfying

$$|\tilde{s}(\alpha) - s(\alpha)| \le \eta$$

on the declared successful preparation class.

It also supplies a finite preparation compared with *τ_{σ}(γs̃)*, with error *e_prep* at its declared complete preparation boundary, including promised residuals and references. Subsequent channels have a complete comparison allowance *e_comp*.

This is not a construction of an unknown-field-to-register channel. The parameter α is a prescribed classical label.

On a supplied qubit *Q* and surviving *A*, take

$$\mathscr{I}_F(X) = r\mathrm{Tr}_Q X, \qquad \mathscr{I}_+(X) = (1-r)\langle 1|X|1\rangle, \qquad \mathscr{I}_-(X) = (1-r)\langle 0|X|0\rangle,$$

where contractions act only on *Q*. The recorded instrument is CPTP, consumes *Q*, and becomes *Tr_Q* after its record is discarded. On the ideal signal,

$$P_s(F) = r, \qquad P_s(\pm) = \frac{1-r}{2}\Big(1 \pm \tanh\frac{\gamma s}{\sigma}\Big).$$

The failure branch is retained, not postselected. These are the signal and clock interface conventions documented in X-J-ST, X-K-ST, and I-B-ECHO [CheungXJ,CheungXK,CheungEcho].

Proposition. Let

$$B_J = \max_J |q_0(\alpha)|, \qquad e = e_{\mathrm{prep}} + \frac{|\gamma|\eta}{2\sigma} + e_{\mathrm{comp}}.$$

Under Assumption ,

$$\mathrm{TV}\,(\widetilde{P}_{\alpha_1}, \widetilde{P}_{\alpha_2}) \geq [\frac{(1-r)|\gamma|}{2\sigma}\mathrm{sech}^2\Big(\frac{|\gamma \mathrm{rvertB}_J}{\sigma}\Big)[\frac{\pi}{2}|\alpha_1 - \alpha_2| - 2E_J]_+ - 2e]_+.$$

The stronger short-gate slope in Proposition may replace its continuum gap. If such records constitute the complete accessible output of an unknown-code task, Theorem still applies.

Proof. The ideal laws satisfy

$$\mathrm{TV}\,(P_{s_1}, P_{s_2}) = \frac{1-r}{2}|\tanh(\gamma s_1/\sigma) - \tanh(\gamma s_2/\sigma)|.$$

Since *|s_j| ≤ B_J*, the mean value theorem and give the ideal lower bound.

Write

$$|e_\sigma(z)\rangle = \cos\theta(z)|0\rangle + \sin\theta(z)|1\rangle.$$

Then *θ^{'}(z) = sech (z/σ)/(2σ)*, so

$$d_{\mathrm{tr}}(\tau_\sigma(z), \tau_\sigma(z^{'})) \leq \frac{|z - z^{'}|}{2\sigma}.$$

Thus each implemented law is within *e* of its ideal law. The triangle inequality subtracts *2e*. The final assertion follows from the classical output type, not the size of the geometric gap.

par *Interpretive boundary*. The statement concerns declared finite channels and, where used, free operators, kernels, and supplied preparations on prescribed continuum data. Destructive Enter does not acquire an unknown continuum functional, select the host geometry, or turn a conserved tensor into a source. No geometry update, Einstein dynamics, or spacetime emergence is inferred.

Remark. The family *α ↦ P_{s(α)}* concerns prescribed classical parameters. Its nonlinear ratio is not asserted to be an affine CPTP map on arbitrary quantum states. Physical acquisition would require its own field representation, coupling, output algebra, calibration, and complete error theorem.

Front asymptotics and spin-domain boundaries

Off shadow lines, gives

$$\Phi_0(0,\theta) = \frac{\sin(\pi/\alpha)}{\cos(\theta/\alpha) - \cos(\pi/\alpha)}.$$

For *Δz = 0*, in the convention *G(k) = ∫ e^{ikt}G(t) dt*,

$$\widehat{G}_{\mathrm{dif}}(k) = -\frac{e^{\mathrm{i}kL + \mathrm{i}\pi/4}\Phi_0(0,\theta)}{4\pi\alpha\sqrt{2\pi r_e r_r L}}k^{-1/2} + O(k^{-3/2}).$$

To see the normalization, near *t = L + s*,

$$G_{\mathrm{dif}}(L+s) = s^{-1/2}h(s), \qquad h(0) = -\frac{\Phi_0(0,\theta)}{4\pi^2\alpha\sqrt{2Lr_e r_r}}.$$

The neutral closed weight is even analytic in *u* near zero, so *h* is analytic in *s*. A cutoff at *s = 0*, the endpoint

Fourier integral

$$\int_0^\infty e^{\mathrm{i}ks} s^{-1/2}\,\mathrm{d}s = \sqrt{\pi} e^{\mathrm{i}\pi/4} k^{-1/2},$$

and the next endpoint term give the local expansion. The cutoff complement is handled by integration by parts; the closed weight gives integrable time derivatives at infinity. These are standard endpoint asymptotic operations [Olver,CheungEcho]. The remainder is additive, including when the leading coefficient vanishes.

The supplied twisted-front formula remains a comparison in I-B-ECHO [CheungEcho]. No charged phase-acquisition apparatus is inferred from it.

For a photon or graviton analogy, scalar formulas cannot replace the operator domains. The supplied I-B-G-S excerpts report the source-test Maxwell–Friedrichs intertwining with a compatible scalar ghost, and a componentwise Friedrichs spin-2 defect in exceptional channels for $1/2 < \alpha < 1$ [CheungIBGS]. Those statements are not detector theorems.

The supplied I-B-G-S-B description reports positive tensor-core Friedrichs selection and exclusion of the distinct curvature-regular idealized branch in its exceptional neutral spin-2 setting [CheungTensor]. No full independent tensor audit is asserted.

Proposition. Suppose a specified family of self-adjoint operators, on common identified Hilbert spaces, has strong-resolvent limit A_F. If another extension $A_{cr} \neq A_F$, the same family cannot also have strong-resolvent limit A_{cr}. Independently, any finite classical-output code channel formed using either selected extension has the obstruction .

Proof. A strong resolvent limit is unique. Equality of one nonreal resolvent determines the self-adjoint operator. The second assertion is Theorem , whose constant is independent of the continuum extension.

This does not prove convergence of an unspecified tensor family. Strong-resolvent convergence alone also does not justify passing an unbounded de Donder or Ward operation to the limit. Analytical target-coefficient enclosures, angular tails, complex-frequency estimates, and differentiated-kernel errors are not a certified full-kernel run or a physical Einstein–matter Hessian [CheungTensor].

Vacuum-polarization and Hadamard questions likewise require their own state and extension data. A scalar one-point function is not a spin-2 tensor channel or a source law [CheungVP].

Finite-precision continuum indistinguishability

Fix $0 < \alpha < 1$, $\zeta \geq 0$, and smooth neutral profiles

$$\alpha \leq p_\varepsilon \leq 1, \quad p_\varepsilon' \geq 0, \quad p_\varepsilon = \alpha \text{ near } 0, \quad p_\varepsilon = 1 \text{ for } r \geq \varepsilon.$$

On the filled spatial manifold prescribe

$$\begin{aligned} q_\varepsilon &= p_\varepsilon^2 \mathrm{d}r^2 + \alpha^2 r^2 \mathrm{d}\varphi^2 + \mathrm{d}z^2, \\ B_\varepsilon &= -\Delta_{q_\varepsilon} + \zeta R_\varepsilon, \qquad R_\varepsilon = \frac{2p_\varepsilon'}{r p_\varepsilon^3} \geq 0, \end{aligned}$$

using nonnegative closed forms. The metric is smooth in proper Cartesian coordinates near the axis. Let $B_0 = A_{F,0}$.

Take fixed real compactly supported smooth exterior profiles h_e, h_r, with radial supports bounded below by $a_e, a_r > 0$. Define

$$\begin{aligned} Y^\varepsilon(t) &= \int_0^t f(t') \langle h_r, \frac{\sin((t-t')\sqrt{B_\varepsilon})}{\sqrt{B_\varepsilon}} h_e \rangle \,\mathrm{d}t', \\ y^\varepsilon[\chi] &= \int_0^T \chi(t) Y^\varepsilon(t)\,\mathrm{d}t, \qquad \chi \in L^1([0,T]). \end{aligned}$$

For complex profiles, real and imaginary task components must be declared separately.

Theorem. For each family –, the exterior identifications give

$$Y^{\epsilon} \to Y^{0} \quad \text{uniformly on every} [0, T].$$

There is exact agreement for

$$0 \le t < a_e + a_r - 2\varepsilon, \qquad \varepsilon < \min(a_e, a_r).$$

Every fixed finite list of window values is consequently approximated arbitrarily closely by smooth cores distinct from the cone. Suppose the ideal finite readout is prepared continuously from this record vector: for example, as a tensor product of finitely many states *τ_{σ}(γy^{ε}[χ])*, with fixed ancillas, followed by one fixed finite complete instrument. Its joint classical law converges in total variation to the cone law. The same is true for any fixed number of independent repetitions.

Within this fixed-profile, continuous-preparation class, no test with a fixed finite budget uniformly distinguishes the cone from all sufficiently small cores with equal-prior success bounded above *1/2* by a fixed positive margin.

Proof. Appendix proves strong-resolvent selection, uniform response convergence, and pre-core agreement.

The finite record vector converges componentwise. Continuity of , telescoping tensor products, and contractivity under the common instrument imply convergence of the *joint* law. Continuous marginals alone would not determine arbitrary separately chosen joint preparations, which is why the joint preparation rule is specified.

For *n* independent repetitions,

$$\mathrm{TV}\,(P_{\varepsilon}^{\otimes n}, P_0^{\otimes n}) \le n\,\mathrm{TV}\,(P_{\varepsilon}, P_0) \to 0.$$

The optimal equal-prior success probability for two classical laws is *(1 + TV (P_{ε}, P_0))/2*, by choosing the event on which one law exceeds the other. This proves the test conclusion.

par *Interpretive boundary*. The statement concerns declared finite channels and, where used, free operators, kernels, and supplied preparations on prescribed continuum data. Destructive Enter does not acquire an unknown continuum functional, select the host geometry, or turn a conserved tensor into a source. No geometry update, Einstein dynamics, or spacetime emergence is inferred.

This is not a theorem that finitely many exact real functionals are non-injective. Arbitrarily close values need not be equal. It is also not a point-source limit or a spatially smeared deficit-separation theorem.

With implementation allowances *e_{ε}*, *e_0*,

$$\mathrm{TV}\,(\widehat{P}_{\varepsilon}, \widehat{P}_0) \le \mathrm{TV}\,(P_{\varepsilon}, P_0) + e_{\varepsilon} + e_0.$$

A larger uncontrolled apparatus could report extra information. A deterministic rounding code can jump at a threshold; convergence of that code requires a margin from the threshold.

The scalar proof matches the restricted neutral setting compared in I-B-ECHO and I-B-VP-5 [CheungEcho,CheungVP]. A profile-uniform explicit rate would require further mode and frequency control. The resolvent-to-record and Bessel matching interfaces in I-B-ECHO identify that obligation without supplying it here [CheungEcho]. Twisted-core and spin-2 echo extensions remain OPEN in this manuscript.

A finite observer can receive reports that become arbitrarily close for a singular cone and smooth small-core models. That is a limitation of this fixed-resolution observation scheme, not proof that every microscopic interior is identical or that exact continuum data could never distinguish them.

The core is a separately specified geometric filling, not the mechanism of a physical horizon shredder. Positive tensor-core selection is another problem again; it is not this scalar echo instrument.

Entropy of an append-only report

Let *R_n = (X_1, …, X_n)* be the whole retained classical history, including failures and stop symbols. At each step, a continuation may depend on the existing history and may act on a quantum residual. ``Append-only'' means that old labels are preserved and one new label is appended. Fix one initial state and the unconditioned law.

Every finite append-only process satisfies

$$H(R_{n+1}) - H(R_n) = H(X_{n+1} \mid R_n) \ge 0.$$

Equality holds exactly when *X_{n + 1}* is almost surely a function of *R_n*. Neither independence nor a TV lower

bound is required.

The diagonal states representing this same history have von Neumann entropy *H(R_n)*, which cannot turn over while that entire append-only classical output is retained.

Proof. Trace preservation of the continuation preserves the old-history marginal. Apply the Shannon chain rule. Finite classical conditional entropy is an average of nonnegative entropies, zero exactly for point-mass conditional laws. The eigenvalues of a diagonal density matrix are its classical probabilities.

par *Interpretive boundary*. The statement concerns declared finite channels and, where used, free operators, kernels, and supplied preparations on prescribed continuum data. Destructive Enter does not acquire an unknown continuum functional, select the host geometry, or turn a conserved tensor into a source. No geometry update, Einstein dynamics, or spacetime emergence is inferred.

This is the precise ``growing pile of strips'' statement. The horizon comparison of Section must not replace quantum radiation entropy by this transcript entropy [Watrous,Page1993].

The theorem does not concern a selected marginal, lossy summary, postselected ensemble, or conditional entropy given later evidence. Updating an observer's information can reduce a conditional entropy without editing the physical log.

Example. For two quantum carriers in $|\Phi_2\rangle = (|00\rangle + |11\rangle)/\sqrt{(2)}$,

$$S(B_1) = 1, \qquad S(B_1B_2) = 0.$$

If both are measured in that basis and the outcomes are appended,

$$H(X_1) = 1, \qquad H(X_1, X_2) = 1.$$

Negative quantum conditional entropy permits the first decrease; classical conditional entropy cannot produce the same decrease.

The pile can grow even when each new strip repeats what was already known. It can also carry increasingly useful classical information while its total Shannon entropy remains non-decreasing. A Page-type question asks about the quantum radiation state under a complete dynamical model [Page1993], not the entropy of this same diagonal pile.

Theorem therefore neither derives a Page curve nor rules out one for a different, unmeasured quantum output. It rules out identifying the two quantities without a comparison theorem.

Resources and thermodynamics are separate specifications

The ledgers

A finite implementation should declare at least

L = (allocated, consumed, and returned names;
peak live and external quantum storage;
classical storage, controls, locations, and duration;
energy boundary and apparatus restoration requirements).

Name counts are not energies. A global live-qubit budget and a scheduled laboratory peak can have different accounting conventions [CheungEnter,CheungIB].

A heat/work calculation additionally specifies Hamiltonians, reservoir states, controls, correlations, and endpoint interaction energies. The following standard thermal identity is included to fix precisely the claim supported by those data [Landauer,ReebWolf].

Let M contain all non-reservoir degrees of freedom in a finite implementation step. Let B be a finite reservoir, with Hamiltonian H_B, initially in

$$\tau_B = \frac{e^{-\beta_{\mathrm{th}} H_B}}{Z_B}, \qquad \beta_{\mathrm{th}} = (k_{\mathrm{B}} T)^{-1}, \quad T > 0.$$

Assume the complete initial state is $\rho_M \otimes \tau_B$ and the complete step is unitary:

$$\omega_{MB} = U(\rho_M \otimes \tau_B)U^{\dagger}.$$

Define

$$Q_B = \mathrm{Tr}\,[H_B(\omega_B - \tau_B)].$$

Using natural-log entropy *S_n*, mutual information *I_n*, and relative entropy *D_n*,

$$\beta_{\mathrm{th}} Q_B = S_{\mathrm{n}}(\rho_M) - S_{\mathrm{n}}(\omega_M) + I_{\mathrm{n}}(M:B)_\omega + D_{\mathrm{n}}(\omega_B \| \tau_B).$$

In particular, a strictly positive complete non-reservoir entropy decrease gives

$$Q_B \geq k_{\mathrm{B}} T (S_{\mathrm{n}}(\rho_M) - S_{\mathrm{n}}(\omega_M)) > 0.$$

The abstract Enter identity does not fix these data or imply that entropy decrease. It yields no universal strictly positive heat cost per readout.

Proof. The Gibbs formula gives

$$D_{\mathrm{n}}(\omega_B \| \tau_B) = \beta_{\mathrm{th}} Q_B - S_{\mathrm{n}}(\omega_B) + S_{\mathrm{n}}(\tau_B).$$

Unitarity and initial independence give

$$I_{\mathrm{n}}(M:B)_\omega = S_{\mathrm{n}}(\omega_M) + S_{\mathrm{n}}(\omega_B) - S_{\mathrm{n}}(\rho_M) - S_{\mathrm{n}}(\tau_B).$$

Eliminate the bath entropy difference. Mutual information and relative entropy are nonnegative. Finally, contains none of the Hamiltonian, temperature, complete unitary, or entropy-decrease specifications. It cannot determine their heat bill.

par *Interpretive boundary*. The statement concerns declared finite channels and, where used, free operators, kernels, and supplied preparations on prescribed continuum data. Destructive Enter does not acquire an unknown continuum functional, select the host geometry, or turn a conserved tensor into a source. No geometry update, Einstein dynamics, or spacetime emergence is inferred.

This theorem is the conditional content of the ``heat'' entry in Section , not an exception to that entry.

Corollary. Under Theorem , suppose $M = YR$, initially classical, with R retained with the same law and the final state classical on R. Let $\dim Y = m \geq 2$. If each supported final conditional state of Y has probability at least $1 - \delta$ in a fixed reset state $|0\rangle$, where $0 \leq \delta \leq 1 - 1/m$, then

$$Q_B \geq k_{\mathrm{B}} T [\,(\ln 2) H(Y \mid R) - h_{\mathrm{n}}(\delta) - \delta \ln (m-1)\,],$$

where $h_n(x) = -x \ln x - (1-x) \ln (1-x)$. The bracket must be positive for this lower bound to imply strictly positive heat.

Proof. Dephase in a basis containing $|0\rangle$; entropy cannot decrease under that pinching. Maximizing the entropy on the orthogonal $(m-1)$-dimensional subspace gives

$$S_{\mathrm{n}}(Y) \leq h_{\mathrm{n}}(\delta) + \delta \ln (m-1).$$

The right side increases on the stated interval. Average over R, cancel its unchanged entropy, and apply .

No exact-reset attainability claim is made for every finite full-rank thermal bath. The corollary is conditional on a process meeting its assumptions [ReebWolf].

Example. Let $H_M = H_B = 0$, $\dim M = \dim B = d$, and $\tau_B = I/d$. SWAP sends

$$\rho_M \otimes I_B/d \,\mathrm{longmapsto}\, I_M/d \otimes \rho_B$$

and has $Q_B = 0$. If only M is subsequently returned through a classical readout, the input is unavailable at that interface. Quantum access to B returns it exactly. The example relocates information; it does not erase the complete input with compulsory positive heat.

The same reported channel can also be accompanied by input-independent driven operations on an extra reservoir, changing the heat bill without changing the report. If interactions vanish at the endpoints and declared control work is the only external supply, the first-law convention is

$$W_{\mathrm{on}} = \Delta E_M + Q_B.$$

This is not a black-hole mass-loss equation.

A physical implementation may heat its surroundings while it resets working memory or restores fresh blanks. When the assumptions of Theorem and a positive entropy decrease hold, the reservoir cost is quantitative.

That does not mean every strip must have a strictly positive universal heat price. A known-state task, available side information, a noncyclic blank supply, or information transferred to a residual changes the physical account. Calling an interface destructive does not settle those choices.

Exterior classical reports and heat access

Definition. For an unknown code Q, a complete classical exterior cut is an accessible output algebra whose state is entirely a finite classical history R, including all declared outcome, timing, failure, control, and heat-meter labels.

A deterministic energy ledger computed from the history adds no new access. A separate finite heat-meter outcome is included in the joint history. No quantum bath, receiver memory, shared entanglement across the cut, or other message-bearing quantum factor is available to the decoder.

For every finite adaptive process with a complete classical exterior cut and unknown d-dimensional input code,

$$\inf_{\mathscr{D}} d_{\diamond}(\mathscr{D}\mathscr{C}_{\mathrm{ext}}, \mathrm{id}_Q) \geq 1 - \frac{1}{d}.$$

This includes joint classical heat labels, correlations between reports, arbitrary finite precision, and arbitrary finite classical postprocessing.

The bound persists for a diamond-norm limit of decoded finite-prefix channels, when such a limit exists. For an implemented channel completely within δ of the classical-cut comparison, the lower bound is $[1 - 1/d - \delta]_{+}$.

Proof. Lemma represents the entire accessible history as one POVM channel. Apply Theorem . All classical postprocessing is absorbed into the decoder.

For a diamond-norm limit of decoded prefix channels, their normalized Choi states converge in trace norm. Each is separable across the finite reference/output split, and that separable-state set is closed. The Bell-overlap argument therefore persists. The implementation allowance is Corollary .

par *Interpretive boundary.* The statement concerns declared finite channels and, where used, free operators, kernels, and supplied preparations on prescribed continuum data. Destructive Enter does not acquire an unknown continuum functional, select the host geometry, or turn a conserved tensor into a source. No geometry update, Einstein dynamics, or spacetime emergence is inferred.

This is the exact obstruction used in the horizon rephrasing of Section . Thermality is not its premise. A nonthermal classical report has the same obstruction, whereas a thermal-looking marginal alone does not prove a classical cut.

An energy expectation, a classical energy-outcome law, and a full quantum reservoir channel are different access objects. Example explains why the distinction cannot be removed.

Proposition. For a finite isometry $V: Q \to Z$, the complete channel $\rho \mapsto V\rho V^{\dagger}$ has a reference-preserving CPTP left inverse. Its classical-only restriction need not have one.

Proof. Let $P = VV^{\dagger}$, and fix a state τ_Q. Define

$$\mathscr{R}(X) = V^{\dagger}XV + \mathrm{Tr}\,[(I - P)X]\tau_Q.$$

Both terms are CP. Their trace effects sum to I_Z, so $\mathscr{R}$ is CPTP. It returns ρ from $V\rho V^{\dagger}$, including with references. The classical restriction is subject to Theorem .

This elementary global/access distinction is consistent with the complete-output formulations in the supplied Hawk04 and En07 records [CheungHawk04A,CheungEn07]. It is not a replacement for the assumptions of exact error-correction or no-hiding theorems [KnillLaflamme,BraunsteinPati].

Proposition. There is a channel from one unknown qubit to a classical key K and a quantum carrier B such that both individual marginals are input independent but their joint output permits exact referenced recovery. There is also a replacement channel with the same two individual marginals and no such recovery.

Proof. Let *P_ab = X^aZ^b, a, b ∈ {0, 1}*, and define

$$\mathcal{M}(\rho) = \frac{1}{4}\sum_{a,b} |a,b\rangle\langle a,b|_K \otimes P_{ab}\rho P_{ab}^{\dagger}.$$

The key marginal is *I_K/4*. Pauli averaging makes the carrier marginal *I_B/2*. Applying *P_ab^{†}* controlled by the key and discarding the key returns ρ exactly, by an operator identity also valid with references.

The replacement channel

$$\mathcal{N}_{\mathrm{rep}}(\rho) = I_K/4 \otimes I_B/2$$

has the same individual marginals and is constant on all input states. No decoder can make its output the identity channel on the qubit.

The recoverable channel in is classical–quantum, not classical-only. It is the familiar private-quantum-channel mechanism [PrivateChannels], used here as an exact counterexample rather than a gravitational model.

A pile may look locally random while its joint quantum structure, together with an available key, still returns a code. Replacing that carrier by a classical readout changes the task. Thus the words ``hot," ``thermal," or ``random" cannot replace Definition . The proposition supports no inference about physical Hawking partners or a gravitational endpoint.

Endpoint underdetermination

Fix a finite prefix instrument

$$\mathcal{P}: Q \to R \otimes A,$$

where *R* is the accumulated classical history and *A* is a remaining quantum factor whose terminal ownership has not been specified.

The prefix and the destructive-readout rule do not determine a unique terminal specification. If *dim A ≥ 2*, the following continuations all preserve the old-record law:

• stop, publish *R*, and keep *A* as an inaccessible residual;

• destructively read *A*, append its outcomes to *R*, and return no live *A* name;

• publish *R* and grant quantum access to *A*. The third is not a classical-only output. Different ownership declarations can distinguish interfaces even when their complete unrestricted tensor-factor maps coincide.

After an actual complete classical cut, no later classical-only continuation can universally recover an unknown code. Nevertheless, the readout rule selects no physical vacuum, remnant, evaporation law, or terminal geometry.

Proof. Each continuation, before restricting ownership, is trace preserving on *A*. Summing or tracing its new outputs leaves *Tr_A 𝒫(ρ)* on the old record *R*.

The specifications differ in access and, for the second, in appended measurement data. The first and third may have the same complete unrestricted channel while granting different accessible factors. Thus one must not describe ownership alone as a different unitary or CPTP dynamics.

After a complete classical cut, postprocessing followed by fresh state preparation remains measure and prepare. Apply Theorem .

Finally, no terminal Hilbert-space assignment, Hamiltonian, stress law, or geometry update is part of the prefix rule. Even fixed terminal classical labels can be appended in different ways without altering its old-record marginal. The missing physical endpoint cannot be selected by that marginal.

par *Interpretive boundary*. The statement concerns declared finite channels and, where used, free operators, kernels, and supplied preparations on prescribed continuum data. Destructive Enter does not acquire an unknown continuum functional, select the host geometry, or turn a conserved tensor into a source. No geometry update, Einstein dynamics, or spacetime emergence is inferred.

This is the rigorous endpoint statement behind the narrative in Section . It is not a remnant theorem.

Even before the endpoint, the output type alone does not determine an evolution: measuring *Z* and measuring *X* have the same classical type and different laws on $|0\rangle$. Actual channels and schedules must be supplied.

A residual holding the original arbitrary quantum code cannot be inferred after that code has already crossed a complete classical cut. A late quantum return requires that some appropriate quantum resource never be excluded, or that a different interface be specified. Missing profile-uniform echo rates and spin-2 echoes are separate analytical obligations, not the proof of endpoint underdetermination.

Long phases, quantum computers, and ``does it explode?''

What can shrink during the proposed long phase

The running-machine story imagines a sequence of inputs entering the mouth. The public transcript grows, and some finite supply of live names may decrease. If a physical implementation also consumes a finite energy reserve while restoring apparatus memory, its energy ledger can decrease as well.

These are separate assertions. The conical host, including α, does not shrink or change during this process. No black-hole mass variable is supplied by the count of names. Fresh allocation can also replenish live names, so even semantic depletion requires an explicit no-allocation condition.

Proposition. Suppose a process begins with N_0 live qubit names, allocates no new names, and its n-th readout consumes $m_n \geq 1$ names. Other stages preserve the number of live names. Then

$$N_n = N_0 - \sum_{j=1}^{n} m_j \geq 0.$$

There are at most N_0 such nonempty readout stages. If every initial name is eventually consumed, the returned live quantum name set is empty.

Proof. At each readout, exactly m_n names are removed. Induction gives . Since every $m_n \geq 1$, the number of stages is at most N_0.

This is a finite resource count, not an energy, temperature, mass, or lifetime formula [CheungEnter,CheungIB].

To say that outgoing energy is paid from a body or a mass reserve, one must add a same-model energy balance. To say that the lost reserve appears specifically as heat requires a further thermodynamic identification.

Theorem can quantify a specified reset with an initially thermal independent reservoir. It does not equate the bath's energy gain with a black hole's mass loss, derive an outgoing luminosity, or imply a metric update. The long-phase analogy is therefore a sequence of declared reports and optional physical ledgers, not derived evaporation dynamics.

The same rule inside a quantum computer

Imagine a computer whose message-bearing quantum degrees of freedom are eventually processed only through destructive Enter readouts. The visible output then consists of finite words, stochastic symbols, failures, and any declared classical energy readings. If no quantum code-bearing factor is returned, its output is subject to Theorem .

This is a deliberately restricted computer. An ordinary quantum computation can include unitary gates, live quantum memory, coherent resource states, and readouts of ancillas while preserving a logical code elsewhere. The mere presence of measurements does not force the whole computation through a classical cut. I-B and the clock-composition records retain such complete resource and residual distinctions [CheungIB,CheungXK].

If every live name is consumed and no quantum output is returned, the declared accessible endpoint holds classical data. A physical implementing device may additionally contain a bath, controller, inaccessible correlations, and un-restored memory. When the thermal assumptions prove heat transfer, that transfer belongs to the complete physical account.

Thus ``classical data plus dissipated heat remain'' is a conditional description of a particular implementation. It is not a theorem that the whole apparatus or environment has become a classical object. Theorem identifies only the persistent accessible output algebra.

Heating is not automatic

The familiar picture of a shrinking quasi-static Schwarzschild black hole becoming hotter belongs to a separately specified Hawking comparison [Hawking1975]. It is not a temperature law for the fixed conical product.

A laboratory apparatus has a different heat capacity and cooling problem. Positive heat delivered to an external

bath need not heat the computer itself. Even positive power dissipated inside a device can be balanced by cooling. No temperature rise follows from Theorem , the consumption count, or the channel identity.

Proposition. Assume, independently of the quantum channel, that a device has a temperature $\Theta(t)$ in an interval I, a differentiable internal-energy function U_dev, and heat capacity

$$C_{\mathrm{dev}}(\Theta) = U'_{\mathrm{dev}}(\Theta) > 0.$$

Suppose its model prescribes the net power balance

$$\frac{\mathrm{d}}{\mathrm{d}t} U_{\mathrm{dev}}(\Theta(t)) = P_{\mathrm{dep}}(t) - P_{\mathrm{out}}(t) = P_{\mathrm{net}}(t).$$

Then, while this model remains valid,

$$\dot{\Theta}(t) = \frac{P_{\mathrm{net}}(t)}{C_{\mathrm{dev}}(\Theta(t))}.$$

Temperature rises, remains constant, or falls according to the sign of P_net.

For a declared threshold $\Theta_fail > \Theta(0)$ in I, it is not reached at times for which

$$\int_0^t P_{\mathrm{net}}(s)\,\mathrm{d}s < U_{\mathrm{dev}}(\Theta_{\mathrm{fail}}) - U_{\mathrm{dev}}(\Theta(0)).$$

Proof. Differentiate $U_dev(\Theta(t))$ by the chain rule. Integrating gives

$$U_{\mathrm{dev}}(\Theta(t)) - U_{\mathrm{dev}}(\Theta(0)) = \int_0^t P_{\mathrm{net}}(s)\,\mathrm{d}s.$$

Strict increase of U_dev in temperature proves .

No explosion is required. If cooling balances deposited power, the conditional thermal model permits constant temperature. If deposited power exceeds cooling, temperature can rise. Reaching a material failure threshold still does not specify whether the device shuts down, melts, burns, or ruptures.

Those outcomes require material, chemical, mechanical, and protection models not supplied here. Trapped heat may motivate an ordinary thermal-failure story, but it is not a new quantum consequence. The mathematical interface demands a classical report at the declared cut; heat is conditional, and explosion is not entailed.

The final stage remains an additional choice

The words ``complete evaporation,'' ``remnant,'' and ``last-moment change'' name possible physical stories, not outputs selected by the finite readout axiom.

Theorem exhibits distinct terminal access specifications. A last-moment quantum return is a change of the classical-only contract. A remnant can hold an unconsumed code only if that code has not already been lost across the complete classical cut. A complete physical evaporation claim requires its own terminal dynamics, geometry, conservation, and residual account.

Profile-uniform echo rates, twisted-core models, and spin-2 echo extensions remain OPEN here. None of them, merely by being solved, would select one of these gravitational endings.

Classical persistence and surviving quantum names

For a finite process whose retained accessible output consists only of append-only classical registers, every accessible persistent observable belongs to their commutative algebra. Every measured name consumed by the process is absent from its returned quantum name set.

If a quantum residual A is returned, the complete output is generally classical–quantum instead. The classical-only obstruction then applies only to its explicitly restricted classical interface, not automatically to the larger output.

Proof. Each destructive primitive has the form . Induction removes the measured names from the returned quantum context and preserves block diagonality of classical labels. If no quantum factor is returned, all permitted observables are diagonal in those labels, hence commute. A returned residual contributes a matrix algebra in each supported classical block, which need not commute.

par *Interpretive boundary*. The statement concerns declared finite channels and, where used, free operators, kernels, and supplied preparations on prescribed continuum data. Destructive Enter does not acquire an unknown continuum functional, select the host geometry, or turn a conserved tensor into a source. No geometry update, Einstein dynamics, or spacetime emergence is inferred.

The theorem is the precise accessible-persistence statement used in Section ; it is not an ontology of the machine.

Example. Let Q carry an arbitrary ρ, and prepare A in $|0\rangle$. SWAP Q and A, then apply destructive Enter to Q. The reported outcome is deterministically *0*, the name Q is consumed, and A contains ρ, including reference correlations. Consumption does not forbid prior logical transfer.

Coherent recording is different again. The isometry

$$W_{\mathrm{rec}} = \sum_j \mathrm{proj}\, j_Q \otimes |j\rangle_C$$

retains quantum carriers and, on superpositions, has off-diagonal joint coherences. It is not by itself a classical-output instrument. The larger nondestructive and smaller destructive interfaces may agree after suitable discards without being the same interface [CheungIB].

Eight open problems reframed, not solved

The information paradox

The shredder rephrasing is useful when it asks whether the complete exterior channel preserves arbitrary code states, including their reference correlations. A classical transcript alone fails that task, even if it reveals many classical parameters.

The missing physical step is a derivation of the actual exterior access algebra and complete channel. A thermal spectrum does not supply it. Theorems and prove the conditional classical-cut obstruction; the physical information paradox is not resolved.

The Page curve and timing of recovery

For an unedited log, new symbols cannot lower the joint Shannon entropy. The timing of quantum information recovery, however, concerns an evolving quantum output and its complete correlations. It cannot be read from the monotonicity of the classical log.

Theorem is the precise transcript result. A gravitational Page history and a coherent radiation decoder remain unconstructed [Page1993].

The final stages of evaporation

An instruction prefix may be complete enough to predict its next declared readout and still say nothing about a physical endpoint. Retaining a residual, consuming it, or granting quantum access are different terminal specifications.

Theorem proves this underdetermination. It is not a proof of complete evaporation, a remnant, or a final information burst.

The measurement problem

Inside the present semantics, a positive-probability branch has a classical outcome and its declared conditional residual. That explains what the interface returns once the Born instrument has been specified; it does not explain why a particular outcome occurs in a chosen interpretation of quantum mechanics.

Proposition and Lemma give the channel and its recovery consequence. No collapse mechanism, branch-selection ontology, or unrestricted measurement-foundation theorem is derived [DaviesLewis,CheungEnter].

The thermodynamic arrow and Landauer erasure

The running shredder suggests irreversible accumulation and dissipation, but these must be separated. An append-only record has a monotone entropy under its own law. A specified entropy-decreasing reset can export heat under the thermal hypotheses.

Theorem and Corollary quantify the latter. They do not derive a global arrow of time, universal heating, or a heat price from name consumption alone [Landauer,ReebWolf].

Whether quantum information is truly lost

The same complete process can preserve a code globally and fail to return it through one chosen factor. The shredder picture is safe when ``lost'' is indexed by that factor and task. It becomes unsafe when a missing accessible decoder is promoted to a universal microscopic destruction law.

Proposition proves global restoration for the complete isometric output, and Theorem proves the classical-access obstruction. Neither decides the microscopic dynamics of an evaporating black hole [BraunsteinPati,CheungHawk04A].

Interior and horizon complementarity

An exterior classical description and an interior quantum residual can sit on opposite sides of a declared access boundary. Their ownership must be stated before a decoder is claimed. The residual is not automatically an independent recoverable copy of the whole code.

Definition fixes the present boundary. No identification of that boundary with physical horizon complementarity, no infalling observer model, and no compatibility theorem between such observers is constructed.

Classicality of the observed world

The accumulating report is classical because the declared observable output algebra is commutative. This may motivate an image of a classical public layer produced by ongoing quantum processes.

Theorem proves only that interface statement. It does not show that all observable physics belongs to one commutative algebra, eliminate quantum residuals, identify a fundamental substance, or derive classical spacetime from records.

Source separations, release, and remaining obligations

Comparison ledger

begingroup toprule Source comparison Documentary connection Non-transfer boundary

midrule endfirsthead toprule Source comparison Documentary connection Non-transfer boundary

midrule endhead bottomrule endfoot

Enter article; I Names, finite destructive semantics, and characterization in a specified class [CheungEnter,CheungI]. No abbreviated rederivation of the Born law or field instrument.

Philosophy; En10-A Readout/recovery, inference, resources, and identification distinctions [CheungPhil,CheungEn10]. Orientation and supplied comparisons, not theorem premises.

I-B; En09-A Prescribed host, clock calibration, complete records and residuals [CheungIB,CheungEn09]. No extraction of geometric time, heat bill, or host metric.

I-B-G Extension-dependent free Green theory, images, diffraction, and finite propagation [CheungIBG]. No unknown-field acquisition or compact-spacetime exactness beyond its declared domain.

I-B-G-S Source-test Maxwell compatibility and exceptional spin-2 domain distinctions [CheungIBGS]. No unrestricted photon detector or physical graviton Hessian.

I-B-G-S-B Supplied positive tensor-core and enclosure description [CheungTensor]. No full independent audit, certified full-kernel run, or Einstein–matter identification.

I-B-ECHO-v2 Kernel, windows, supplied finite reports, neutral cores, and resolvent/matching interfaces [CheungEcho]. No geometric reconstruction, global shadow-uniform resolution, or exact-real non-injectivity theorem.

X-J-ST; X-K-ST Complete signal/clock instruments, composition, resources, and complete errors [CheungXJ,CheungXK]. No physical acquisition, automatic causal embedding, or unpassed resource-fit certificate.

I-B-VP Supplied infrared, Hadamard, one-point, and extension-ledger comparisons [CheungVP]. Covariances, spin tensors, detector channels, and sourcing are not identified.

Hawk; En07 Global preservation, restricted recovery, complete residuals, and endpoint conditions

[CheungHawkClassical,CheungHawk04A,CheungEn07]. No physical evaporation result from a finite code calculation.

XIV/XV/XVI Version-specific analytical stops and scoped releases [CheungAnalyticStops,CheungXVI]. No missing characteristic composition, quantum Ward datum, or inherited admission supplied by readout.

OpenAI-attributed catalogues Supplied documentary records [CheungCatalogue]. No family verified or adopted as a theorem premise.

endgroup

The analogy does not alter any row of this ledger.

Standing programme and evidence status

The standing status retained in this manuscript is

$$\mathrm{O.1} = \texttt{texttt}\, \mathit{NOT_CLOSED}.$$

I-C does not discharge XVI-N or activate the downstream admission exit called XV-B.

XIV-H's historical H2 stop is not erased, and XIV-I's narrower constructed I2 internal-amplitude result is not withdrawn. The required full maps at I3/J2, subsequent section, transverse estimates, contact compatibility, output wavefront theorem, and inherited quantum admission are not supplied here [CheungAnalyticStops]. A technical sheet named XV-B may exist while the downstream admission exit remains unopened.

The supplied XVI-Q record contains a separately scoped two-point closure under a prescription different from XVI-O's. I-C neither independently audits that release nor converts it into a blanket complete-closure or complete-failure statement [CheungXVI].

No new discharge is claimed for inherited problem numbers. In the supplied programme terminology, Problems 1, 2, and 8 are not completed here; Problems 3–7 and 9–11 are not reached; the distinct Problem 6 is not this implication; and Problem 12 is not resolved. These are documentary non-transfer statements, not additional mathematical theorems.

Numerical status. The legacy I-B-ECHO grid report

$$\max_{\text{supplied grid}} \frac{|s - q_0|}{\varepsilon_W} = 0.252$$

remains SUPPLIED and UNCERTIFIED. It is not replaced by an exact *0. 25* claim and is not used in any proof. No new numerical calculation, Page plot, or tensor full-kernel certificate is reported in this expansion.

Mathematical release

Proposition. Under the printed hypotheses:

• Theorems and give the sharp referenced classical-cut recovery obstruction, including classical heat labels and complete implementation allowances.

• Propositions and give geometric report separation under the declared scalar kernel and supplied preparation.

• Theorem gives finite-budget indistinguishability for fixed smeared reports and continuous finite preparations, not non-injectivity of exact real functionals.

• Theorem proves append-only transcript entropy monotonicity, not a quantum Page history.

• Theorem gives a separate conditional thermal balance, not compulsory positive heat per consumed name.

• Theorems and give endpoint underdetermination and classical persistence at the declared output.

• Propositions and state the additional no-allocation and thermal-balance hypotheses needed for semantic depletion and heating claims.

• The vernacular, horizon, and quantum-computer mappings are explicit analogies. They do not constitute physical identifications of the interface, apex, radiation, bath, or endpoint. No physical evaporation, remnant selection, Einstein source law, metric reconstruction, spacetime emergence, physical spin-2 Hessian, or inherited characteristic quantum coefficient is constructed.

Proof. The named finite results are proved locally. The continuum application uses Input statement , the displayed

window algebra, and the neutral-core proof in Appendix . The analogy remarks introduce no physical correspondence as a theorem hypothesis or conclusion. The excluded objects are not constructed by any of the maps or forms in these proofs.

par *Interpretive boundary*. The statement concerns declared finite channels and, where used, free operators, kernels, and supplied preparations on prescribed continuum data. Destructive Enter does not acquire an unknown continuum functional, select the host geometry, or turn a conserved tensor into a source. No geometry update, Einstein dynamics, or spacetime emergence is inferred.

Potential significance and successor requirements

The useful combined certificate is

$$\text{boxed}\begin{array}{c}\text{geometric sensitivity of declared reports}\\ +\ \text{complete finite implementation allowances}\\ +\ \text{classical-cut recovery obstruction}\\ +\ \text{separate physical cost specification.}\end{array}$$

It allows an informative classical experiment to be described without promoting its information into coherent recovery. This is the integration claimed here, not a priority claim for entanglement breaking, state estimation, entropy chain rules, or Landauer balance.

A stronger physical successor would have to supply:

• a field-to-register acquisition channel on a stated input representation, including failures and complete residuals;

• a physical code, local loading, emission, propagation, and coherent receiver interface for a horizon interpretation;

• a complete same-model energy account and, if claimed, a backreaction or geometry-update law;

• a mode/frequency-uniform echo estimate for any explicit profile-uniform core rate;

• domain-compatible twisted, vector, or spin-2 core and echo theorems, rather than scalar relabeling;

• a radiation state and dynamics before any Page-history claim, with access and complete complementary outputs retained;

• a material and thermal-control model before a heating or failure prediction;

• archived computations with total error enclosures before legacy decimal reports are certified. A longer analogy does not discharge any item.

appendix

Proof of the neutral-core statements

The measures satisfy

$$\mathrm{d}V_\varepsilon = p_\varepsilon \mathrm{d}V_0, \qquad \mathscr{H}_\varepsilon = L^2(\mathrm{d}V_\varepsilon).$$

Identify them unitarily by

$$U_\varepsilon : \mathscr{H}_\varepsilon \to \mathscr{H}_0, \qquad U_\varepsilon v = \sqrt{p_\varepsilon}\, v,$$

and put $B_{\varepsilon} = U_{\varepsilon}B_{\varepsilon}U_{\varepsilon}^{-1}$.

Lemma. For every family –,

$$\widetilde{B}_\varepsilon \to B_0$$

in the strong-resolvent sense.

Proof. We check the weak lower-bound and strong recovery conditions for the nonnegative transferred closed forms.

The neutral cone axis has zero local H^1-capacity. For $0 < \delta < 1$, choose a radial cutoff χ_{δ} that is zero on $r \le \delta^2$, one on $r \ge \delta$, and satisfies

$$|\chi_\delta'(r)| \le \frac{C}{r|\ln \delta|}.$$

On a compact *z*-range,

$$\int |\nabla \chi_\delta|^2 \, dV_0 \le \frac{C'}{|\ln \delta|} \to 0.$$

For a bounded compactly supported finite-energy function, multiplication by this cutoff converges in form norm. Truncation, large compact cutoffs, and smooth approximation away from the axis extend the conclusion to the whole finite-energy space. Hence that space is the neutral Friedrichs form domain, without an additional axis trace condition.

Let *w_{ε}rightharpoonup w* in $\mathscr{H}_0$. Pass to a subsequence realizing the finite liminf of the transferred energies; the infinite case is immediate. On each fixed exterior $r > \delta$, for $\varepsilon < \delta$, the measure, operator coefficients, identification, and curvature potential agree with the cone. Local weak *H^1* compactness and lower semicontinuity give the cone-energy lower bound there. Exhaust the exterior and let *δdownarrow 0*. The omitted core form contribution is nonnegative because *ζR_{ε} ≥ 0*. Thus

$$\mathfrak{b}_0[w] \le \liminf_{\varepsilon \,\mathrm{downarrow}\, 0} \widetilde{\mathfrak{b}}_\varepsilon[w_\varepsilon].$$

The capacity argument puts *w* in the Friedrichs form domain.

Conversely, approximate a cone form-domain vector by *w_n ∈ C_c^{∞}* supported away from the axis. For small enough *ε*, the transferred form on *w_n* equals the cone form exactly. A diagonal choice gives a strong recovery sequence.

These are the Mosco conditions. Their variational-resolvent consequence gives the stated convergence [Mosco,ReedSimon]. No bound on derivatives of the shrinking profiles was used.

Lemma. For the fixed exterior profiles in , *Y^{ε} → Y^0* uniformly on every finite interval.

Proof. For $\tau \ge 0$, let

$$\varphi_\tau(\lambda) = \begin{cases} \sin(\tau\sqrt{\lambda})/\sqrt{\lambda}, & \lambda > 0, \\ \tau, & \lambda = 0. \end{cases}$$

It is continuous on *[0, ∞)*, bounded by *τ*, and vanishes at infinity. Strong-resolvent convergence therefore gives strong convergence of *φ_{τ}(B_{ε})* for fixed *τ* [ReedSimon]. The exterior profiles satisfy *U_{ε}h_j = h_j* for small *ε*. Dominated convergence gives pointwise response convergence.

The spectral theorem also gives

$$\|\varphi_\tau(\widetilde{B}_\varepsilon)\| \le \tau, \qquad \|\varphi_\tau(\widetilde{B}_\varepsilon) - \varphi_{\tau'}(\widetilde{B}_\varepsilon)\| \le |\tau - \tau'|.$$

Together with the convolution formula, these bounds make the responses uniformly bounded and Lipschitz on *[0, T]*. The contribution from a varying upper integration endpoint is bounded using *φ_0 = 0* and *‖φ_{τ}‖ ≤ τ*. A finite time-net argument upgrades pointwise convergence to uniform convergence.

Lemma. For *ε < min (a_e, a_r)*,

$$Y^\varepsilon(t) = Y^0(t) \quad \text{if} 0 \le t < a_e + a_r - 2\varepsilon.$$

Proof. Choose a cylinder *r = b > ε* below both profile supports. Both equations agree outside it. A wave from the emitter cannot reach this cylinder before *a_e − b*. A difference generated there cannot reach the receiver before a further *a_r − b*. Finite propagation and local uniqueness in the common exterior give equality for *t < a_e + a_r − 2b*. Let *bdownarrow ε*.

Cone finite propagation is part of the declared input. For the smooth core it follows from the local wave equation; the smooth zeroth-order potential does not enlarge its metric propagation cone.

For every fixed window,

$$|y^\varepsilon[\chi] - y^0[\chi]| \le \|\chi\|_{L^1([0,T])} \sup_{0 \le t \le T} |Y^\varepsilon(t) - Y^0(t)| \to 0.$$

Since *p_{ε}* rises from *α < 1* to one, *p_{ε}^{'}* is not identically zero. Each filling has nonzero curvature somewhere and a smooth axis, and is not the singular cone.

Audit of the six claims and the v2 expansion

toprule Claim Precise result Necessary boundary

midrule A Referenced error at least $1 - 1/d$; Haar-average infidelity at least $(d - 1)/(d + 1)$. Code, metric, input task, and receiver resources are fixed.

B $H(R_{n + 1}) = H(R_n) + H(X_{n + 1} \mid R_n)$. Not unmeasured radiation entropy or conditioned knowledge.

C Thermal entropy identity and conditional reset bound. No universal heat cost per consumed name.

D Complete classical reports, including classical heat labels, cannot universally return an unknown code. No additional quantum bath, memory, or entangled resource.

E A prefix admits distinct terminal specifications. Ownership differences need not be different unrestricted channels; no gravitational ending selected.

F Classical-only accessible outputs have commuting persistent observables. Quantum residuals and prior logical transfer are retained in the full account.

bottomrule

Changes from I-C-v1. This expansion:

• reflects the shredder analogy in the title and supplies a dictionary for names, records, residuals, and heat;

• develops the horizon and quantum-computer stories inside explicit analogy remarks rather than physical identifications;

• states the direct-gate unit-height normalization explicitly;

• specifies a common continuous *joint* preparation rule in the core-record theorem, rather than inferring joint convergence from marginals;

• replaces the ambiguous equatorial two-Bell-code discussion by the exact Pauli-key example ;

• distinguishes terminal ownership changes from changes of the unrestricted CPTP map;

• adds no-allocation depletion and conditional thermal-balance propositions so that running out and heating are not attributed to readout alone;

• expands the eight open-problem rephrasings without declaring their physical solutions.

No relative asymptotic is formed from a zero front coefficient. No zero calibration is divided into a ratio. No smoothing operation is assumed safe without preserving support, isolation, and positive overlap. No finite instrument is said to acquire the continuum functional.

Declarations

Nature of the work. This is an analytical finite-channel study with a conditional free-kernel continuum application. No empirical dataset is used.

Source status. The supplied full I-C-v1 and I-B-ECHO-v2 sources, supplied manuscript excerpts, public documentary pages, and summary-level descriptions are distinguished from journal publications and from full independent proof audits. The public Enter essay and I-B page were consulted for documentary background. An unavailable full tensor-core source is not reconstructed from labels.

Verification and submission boundary. The source is intended as one pdfLaTeX file with an internal bibliography and no external figures or data. No successful pdfLaTeX execution, numerical or symbolic run, mechanized proof, independent technical review, physical calibration, experiment, registration, or journal acceptance is asserted. Actual compilation, metadata confirmation, and archival availability of version-qualified unpublished references remain submission tasks.

Physical scope. No gravitational source law, Einstein equation, evolving metric, Hawking channel, Page curve, complete evaporation, remnant selection, physical spin-2 Hessian, or spacetime-emergence result is constructed. No inherited characteristic quantum coefficient is admitted.

References

N. Cheung, ``The Enter Calculus: A Linear Quantum Lambda Calculus with an Operational Characterization of Measurement," *Frontiers in Physics* **14** (2026), article record 1954943. DOI: 10.3389/fphy.2026.1954943. Publisher accepted-article record; accepted 28 September 2026. No later formatted-edition status is inferred from that date. https://www.frontiersin.org/journals/physics/articles/10.3389/fphy.2026.1954943/abstract.

N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, EntPhil-v2.tex, version 2. Introductory documentary essay, not a theorem premise. https://enter.grok.me/enter.

N. Cheung, *Landmark I*, version 2. Supplied unpublished manuscript excerpts, including thm:main, ``Characterization." Its declared strategy class is not shortened or re-proved here.

N. Cheung, *En10-A*, version 1. Supplied unpublished excerpts, including ``Motivation and the philosophical background of Enter: Enter as an interface discipline." Parameter inference and quantum-return decoding remain distinct.

N. Cheung, *I-B: A Finite Enter Spacetime: Calibrated Destructive Time Readout on a Prescribed Conical Product* , version 1. Enter Calculus Companion, unpublished manuscript record. Comparisons include def:experiment, thm:instrument, prop:interfaces, clock calibration, resource accounting, and thermodynamic boundaries. https://enter.grok.me/browse/I-B.

N. Cheung, *En09-A: Realization of Relational Time by Destructive Enter on a Clock Register*, version 1. Supplied unpublished source record. https://enter.grok.me/browse/En09-A.

N. Cheung, *I-B-G: Intrinsic and holonomy-twisted Green operators on a prescribed conical spacetime: Self-adjoint extensions, diffraction, finite propagation, and supported transfer from flat-completion laboratories*, version 2, source identifier I-B-G-v2.tex. Supplied kernel description and Green-domain excerpts. The scalar kernel used here is an explicit input. The source's thm:green and rem:compact-exact retain the spectral-domain versus compact-spacetime distinction. https://enter.grok.me/browse/I-B-G.

N. Cheung, *I-B-ECHO-v2: Diffracted echoes and finite supplied-report instruments*, version 2. Supplied full source I-B-ECHO-v2.tex, October 2026. Relevant locators: input:kernel, lem:front, thm:gtd, thm:readout, thm:finiteinjectivity, thm:clock, thm:signal, thm:TV, lem:selection, thm:core, cor:nonreconstruction, thm:rateinterface, and prop:matching. Legacy numerical reports remain supplied and uncertified.

N. Cheung, *I-B-G-S: Intrinsic spin-1 and spin-2 Green operators on the prescribed conical spacetime*, version 2. Supplied unpublished excerpts, including ass:host, thm:spin1, thm:spin2, and domain/normalization boundaries. No independent full audit is asserted.

N. Cheung, *I-B-G-S-B*. Supplied programme description of positive tensor-core selection, exceptional source-test defects, matching, and analytical kernel enclosures. Complete version-qualified source was not supplied for an independent proof audit. Comparison locators include prop:warddefect, thm:selection, cor:exclude, thm:contrast, lem:modetail, lem:complexbound, and thm:Cauchy. No unstated tensor theorem or computational certificate is imported.

N. Cheung, *I-B-VP: Vacuum polarization of the Friedrichs ground state on a prescribed conical spacetime*, version 3 supplied excerpts; and supplied *I-B-VP-5* neutral-core descriptions. Unpublished documentary comparisons concerning states, infrared/Hadamard control, one-point observables, extensions, and core selection. The restricted neutral-core proof used here is printed locally.

N. Cheung, *X-J-ST: A second charged coefficient and a selective amplitude package*, version 1. Supplied unpublished excerpts, including thm:signal, thm:clock, Lipschitz/TV comparisons, failure retention, and non-acquisition boundaries. The finite identities used here are proved locally.

N. Cheung, *X-K-ST: Certified composition of destructive clocks and finite instruments*, version 1. Supplied unpublished excerpts, including thm:oneclock, thm:compose, thm:error, thm:noenlarge, def:grade, and thm:release. A complete channel certificate is not a passed physical resource fit.

N. Cheung, *VII-J*, version 1. Supplied unpublished excerpts, especially thm:release. Scheduled ancestors and independently certified metric containment remain distinct.

N. Cheung, *Hawk04-B*, version 2 supplied excerpts, and *Hawk05-A*, supplied programme description. Unpublished classical-diary recovery comparisons. The recovery bound in I-C is proved independently.

N. Cheung, *Hawk04-A: Unitary Information Preservation and Radiation Recovery: A Complete-Complement Formulation of the Black-Hole Information Problem*, version 1. Supplied unpublished excerpts. Global preservation, restricted recovery, and a physical black-hole correspondence are different claims.

N. Cheung, *En07-A*, *En07-B*, and *En07-C*. Supplied unpublished programme descriptions of complete restoration, readout/discard, thermodynamic boundaries, and conditional endpoint obligations.

N. Cheung, *Em03-A*, version 1; *Em03-F*, version 1; and *Em03-G*, version 1. Supplied unpublished excerpts on fixed-background scalar emission and distinct effective exterior-source channels. Their source identification, coherent access, and gravitational interpretation are not transferred to I-C.

N. Cheung, *XIV-H*, version 1; *XIV-I*, version 1; *XIV-J*, version 1; and supplied *XV-B* records. Unpublished supplied excerpts. Relevant locators: XIV-H thm:H2; XIV-I thm:I2, thm:I3; XIV-J prop:descent, lem:null, thm:reference, thm:conditionalextension, thm:J2, cor:noadmission. Documentary non-transfer only.

N. Cheung, *XVI-N*, revision r12; *XVI-O*, revision o70-sync; and *XVI-Q*, release Q11. Supplied unpublished excerpts. The prepared-source obstruction, standing XVI-O NOT_CLOSED scope, and separately scoped Q11 two-point release are not identified.

N. Cheung, *Oai282-A* and related supplied comparison records; and OpenAI-attributed catalogue excerpts supplied with the revision material. Documentary only. Catalogue attribution and underlying claims are not independently verified, and no catalogue theorem is a premise.

E. B. Davies and J. T. Lewis, ``An operational approach to quantum probability,'' *Communications in Mathematical Physics* **17** (1970), 239–260. DOI: 10.1007/BF01647093.

M. Horodecki, P. W. Shor, and M. B. Ruskai, ``Entanglement breaking channels,'' *Reviews in Mathematical Physics* **15** (2003), 629–641. DOI: 10.1142/S0129055X03001709. Preprint title: ``General Entanglement Breaking Channels,'' arXiv:quant-ph/0302031.

J. Watrous, *The Theory of Quantum Information*, Cambridge University Press, 2018. Author manuscript: https://cs.uwaterloo.ca/~watrous/TQI/.

S. Massar and S. Popescu, ``Optimal extraction of information from finite quantum ensembles,'' *Physical Review Letters* **74** (1995), 1259–1263. DOI: 10.1103/PhysRevLett.74.1259.

M. A. Nielsen, ``A simple formula for the average gate fidelity of a quantum dynamical operation,'' *Physics Letters A* **303** (2002), 249–252. DOI: 10.1016/S0375-9601(02)01272-0. arXiv:quant-ph/0205035. The average-fidelity/entanglement-fidelity relation is attributed there to the earlier Horodecki work.

W. K. Wootters and W. H. Zurek, ``A single quantum cannot be cloned,'' *Nature* **299** (1982), 802–803. DOI: 10.1038/299802a0.

C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, ``Teleporting an unknown quantum state via dual classical and Einstein–Podolsky–Rosen channels,'' *Physical Review Letters* **70** (1993), 1895–1899. DOI: 10.1103/PhysRevLett.70.1895.

A. Ambainis, M. Mosca, A. Tapp, and R. de Wolf, ``Private quantum channels,'' in *Proceedings of the 41st Annual Symposium on Foundations of Computer Science* (2000), 547–553. Preprint title: ``Private Quantum Channels and the Cost of Randomizing Quantum Information,'' arXiv:quant-ph/0003101.

R. Landauer, ``Irreversibility and heat generation in the computing process,'' *IBM Journal of Research and Development* **5** (1961), 183–191. DOI: 10.1147/rd.53.0183.

D. Reeb and M. M. Wolf, ``An improved Landauer principle with finite-size corrections,'' *New Journal of Physics* **16** (2014), 103011. DOI: 10.1088/1367-2630/16/10/103011. arXiv:1306.4352.

E. Knill and R. Laflamme, ``Theory of quantum error-correcting codes,'' *Physical Review A* **55** (1997), 900–911. DOI: 10.1103/PhysRevA.55.900. arXiv:quant-ph/9604034.

S. L. Braunstein and A. K. Pati, ``Quantum information cannot be completely hidden in correlations: Implications for the black-hole information paradox,'' *Physical Review Letters* **98** (2007), 080502. DOI:

10.1103/PhysRevLett.98.080502. arXiv:gr-qc/0603046.

D. N. Page, ``Information in black hole radiation,'' *Physical Review Letters* **71** (1993), 3743–3746. DOI: 10.1103/PhysRevLett.71.3743. arXiv:hep-th/9306083.

S. W. Hawking, ``Particle creation by black holes,'' *Communications in Mathematical Physics* **43** (1975), 199–220. DOI: 10.1007/BF02345020.

J. Cheeger and M. Taylor, ``On the diffraction of waves by conical singularities. I,'' *Communications on Pure and Applied Mathematics* **35** (1982), 275–331. DOI: 10.1002/cpa.3160350302.

J. Cheeger and M. Taylor, ``On the diffraction of waves by conical singularities. II,'' *Communications on Pure and Applied Mathematics* **35** (1982), 487–529. DOI: 10.1002/cpa.3160350403.

F. W. J. Olver, *Asymptotics and Special Functions*, Academic Press, 1974; reprinted by A K Peters, 1997. Endpoint Fourier and Watson-type asymptotic estimates.

U. Mosco, ``Composite media and asymptotic Dirichlet forms,'' *Journal of Functional Analysis* **123** (1994), 368–421. DOI: 10.1006/jfan.1994.1093.

M. Reed and B. Simon, *Methods of Modern Mathematical Physics I: Functional Analysis*, revised and enlarged edition, Academic Press, 1980.