I-B-v1: A Finite Enter Spacetime

Calibrated Destructive Time Readout on a Prescribed Conical Product

Ngo Cheung

Version 1

Abstract

We construct a finite operational model on the regular part of an explicit 3 + 1-dimensional conical Lorentzian background. Its additional data are finite quantum names, locations, accessible algebras, a complete control and resource interface, and a calibration from a finite clock basis to geometric time labels. Destructive Enter consumes the clock names and returns a classical code together with the surviving matter. We prove the complete instrument identity, the distinction between event-time calibration and history-label calibration, the absence of a universal quantum return, and a finite-dimensional canonical-commutator obstruction. Scheduled influence is contained in the prescribed metric causal relation. A target-adapted finite chain supplies exact reference-preserving SWAP transport for a prescribed local null displacement, with an explicit resource ledger. Several necessary corrections are incorporated. The conical defect in the product is a worldsheet, not a worldline. An arbitrary clock-basis change is not a spacetime coordinate transformation. A stochastic reading at one fixed event cannot equal several distinct coordinates of that event. A relational lapse cannot replace an entire cosmological metric, and spatial homogeneity does not imply flatness. The construction is a spacetime model with an Enter interface, not emergence of spacetime from records. No inherited quantum response coefficient is admitted and no standing analytic gate is discharged.

Bibliographic record

Title	I-B-v1: A Finite Enter Spacetime
Author	Cheung, Ngo
Publication date	2026
Institution	Enter
Report number	04-I-B-v1
Publisher	Cheung Ngo Medical Limited
First page	1
Last page	21
Language	en

Scope, provenance, and necessary corrections

Enter as an interface discipline

The motivating discipline is Cheung's separation of destructive readout, quantum recovery, physical implementation, and geometric identification. Enter removes measured quantum names from the returned quantum interface and supplies a classical record. This is an access statement, not annihilation of every implementing degree of freedom. Likewise, equality of channels is not a thermodynamic account, and an analogy is not a mathematical dictionary [CheungPhil,CheungI,CheungEn03].

We adopt specified finite Born semantics. We do not claim to rederive Landmark I's characterization from an abbreviated list of requirements. That characterization has its own structural class and hypotheses [CheungI]. All mathematical assertions needed for the model below are stated with their hypotheses and proved locally.

The construction follows En03-A's four-part geometric interface: external classical geometry, locations, accessible coupling algebras, and complete control and resource data [CheungEn03]. Geometry is supplied, not extracted

from a count of names or records.

What ``finite'' means

The background is a continuum. An experiment is finite: it has finitely many active names, finite-dimensional quantum factors, finitely many classical outcomes, and finitely many instructions. Classical geometric parameters may be prescribed real numbers. No assertion is made that arbitrary such numbers have been determined by a finite-bit physical calibration procedure.

The main release chooses the conical branch with unit lapse. A cosmological attachment interface is given in Appendix ; its input geometry is not silently identified with the conical product.

Corrections to the proposed architecture

1.16 toprule Potential identification Correction used here

midrule The product defect is a worldline After adjoining z, the full defect has coordinates (T, z) and is a worldsheet. Fixing z gives one worldline.

Clock outcome equals the time of one fixed event For distinct calibrated labels, exact event-time agreement forces a deterministic clock outcome. Nontrivial history conditioning instead refers to different labelled events.

Changing the clock basis changes time coordinates A general basis change changes the observable. Coordinate relabelling is a much more restrictive operation.

Scheduled past lies in metric future An ancestor of an output lies in its metric past; equivalently, the output lies in the ancestor's metric future.

A lapse jet supplies a cosmological metric A spatial metric and shift are also required. The exact background and the formal perturbation parameter remain distinct.

Spatially constant metric components imply flatness Spacetime-constant components imply flatness. Time-dependent, spatially homogeneous metrics need not be flat.

Enter consumes the geometric time axis Enter consumes a finite clock encoding. The classical manifold, coordinate, and future cone remain present.

Exact null transport follows on a fixed lattice Our exact result uses a declared target-adapted chain. It is not an exact-attainment claim for a previously fixed lattice.

bottomrule

These corrections restrict the theorem to a coherent, nonempty model; they are not additional emergence assumptions.

Standing boundaries. Throughout this paper, ``XV-B is not opened'' means that the inherited downstream admission exit is not activated by this construction. It does not deny the existence of a technical manuscript bearing that identifier. XVI-N is not discharged; O.1 remains NOT_CLOSED for the purposes of this release; no inherited quantum coefficient is admitted; and no compilation, numerical execution, physical implementation, or machine verification is asserted. The narrower internal-amplitude advance in XIV-I is not withdrawn, but it is not the full joint characteristic map absent at I3/J2 [CheungXIVIJ,CheungStops].

The geometric background

The cone, its regular part, and its completion

Fix

$$0 < \mu < 2\pi, \qquad \alpha = 1 - \frac{\mu}{2\pi}, \qquad c > 0.$$

Let C_{μ} be the completion of

$$C_\mu^\circ = (0,\infty) \times (\mathbb{R}/2\pi\mathbb{Z})$$

with spatial line element

$$h_{C_\mu} = \mathrm{d}r^2 + \alpha^2 r^2\, \mathrm{d}\varphi^2.$$

Write *o* for its apex. The circumference of the circle $r = R$ is $(2\pi - \mu)R$, so the opening angle is $2\pi - \mu$.

Set

$$\overline{M} = \mathbb{R}_T \times C_\mu \times \mathbb{R}_z,$$

$$M = \mathbb{R}_T \times C_\mu^\circ \times \mathbb{R}_z,$$

$$h = \mathrm{d}r^2 + \alpha^2 r^2\,\mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

For any smooth positive lapse N on M, prescribe

$$g_N = -c^2 N^2\,\mathrm{d}T^2 + h.$$

The main model uses

$$\boxed{g} = -c^2\,\mathrm{d}T^2 + \mathrm{d}r^2 + \alpha^2 r^2\,\mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

In geometrized units $c = 1$, this is the metric in the proposed conical-product branch. Equivalently, with $\theta = \alpha\varphi$, the angular coordinate has period $2\pi - \mu$ and the angular term is $r^2\,d\theta^2$.

The singular completion locus is

$$\mathscr{W} = \mathbb{R}_T \times \{o\} \times \mathbb{R}_z.$$

It is two-dimensional. For fixed z_0, the subset $\mathscr{W}_z_0 = \mathbb{R}_T \times \{o\} \times \{z_0\}$ is a worldline.

The smooth Lorentzian manifold in our theorem is M, not $\overline{M}$ equipped with a purported smooth metric at its apex. The completion records the conical defect. No smooth chart or ordinary pointwise Einstein tensor is asserted on $\mathscr{W}$.

Proposition. For $N > 0$, g_N has signature $(-, +, +, +)$, and

$$g_N^{-1}(\mathrm{d}T, \mathrm{d}T) = -\frac{1}{c^2 N^2}.$$

Choose the time orientation for which ∂_T is future directed. For

$$v = a\partial_T + w, \qquad w \in T(C_\mu^\circ \times \mathbb{R}),$$

a nonzero future-causal vector satisfies

$$a > 0, \qquad |w|_h \le cNa.$$

A static observer has proper-time increment

$$\mathrm{d}\tau = N\,\mathrm{d}T.$$

In particular, T is proper time up to the chosen origin for the static observers of .

Proof. In the displayed coordinates the metric matrix is diagonal, with one negative and three positive entries. Its inverse gives the stated norm of dT. The causal inequality is

$$g_N(v, v) = -c^2 N^2 a^2 + |w|_h^2 \le 0.$$

A nonzero causal vector cannot have $a = 0$; the chosen orientation therefore gives $a > 0$. Along a static curve, $-g_N/c^2 = N^2\,dT^2$, which proves the proper-time formula.

Local rods and holonomy

On a sufficiently small angular interval choose a real lift of φ. The developing map

$$\mathrm{dev}\,(T, r, \varphi, z) = (T, r\cos(\alpha\varphi), r\sin(\alpha\varphi), z)$$

is a local isometry from to Minkowski space with metric $-c^2\,dT^2 + dx^2 + dy^2 + dz^2$. Its spatial coordinates are three local rods.

Proposition. The metric is locally flat on M. In the convention

$$\mathrm{dev} \circ \mathrm{T}^{-1} = J_\mu \circ \mathrm{dev}, \qquad \mathrm{T}(\varphi) = \varphi + 2\pi,$$

the gluing map $J_\{\mu\}$ fixes T, z and rotates the developed (x, y)-plane by μ. Thus local flatness does not remove the

global conical gluing.

Proof. Differentiating gives

$$\mathrm{d}x^2 + \mathrm{d}y^2 = \mathrm{d}r^2 + \alpha^2 r^2\,\mathrm{d}\varphi^2.$$

For the deck transformation, the developed spatial angle changes by $-2\pi\alpha = \mu - 2\pi$, which is the same rotation as μ.

This geometry is motivated by the conical constructions in VII-E, VII-F, and VIII-B [CheungVIIEF,CheungVIIIB]. The additional z direction and the lapse interface are declared here; they are not conclusions transferred from a $2 + 1$-dimensional theorem.

Remark. VII-F's particle-supported cellulation event is a geometric construction in its own declared family [CheungVIIEF]. We do not import its cell counts or uniqueness conclusion into a three-dimensional spatial cellulation. Our operational events below lie in the regular part. Attaching that separate slicing event would require its full geometric data and a clock-and-record dictionary. A slicing event is not already an Enter event.

The local causal chart used operationally

Choose an open spatial set $U \subset C_{\mu}^{\circ} \times \mathbb{R}$ whose development is an open convex Euclidean set, and write

$$\chi : U \to V \subset \mathbb{R}^3$$

for its spatial developing chart. All finite protocols in the main construction remain inside U.

Lemma. Let $x, y \in U$ and $T_1 \geq T_0$. If

$$\|\chi(y) - \chi(x)\|_2 \leq c(T_1 - T_0),$$

then $(T_1, y) \in J_g^{+}((T_0, x))$. If equality holds and $x \neq y$, the straight developed segment, traversed at constant speed, is null.

Proof. Convexity keeps the straight spatial segment in V. Its lift has constant coordinate velocity of norm at most c, so applies with $N = 1$. Equality makes its tangent null.

No global formula for $J^{\pm}$ across the singular completion is needed.

Finite names, access, control, and destructive Enter

Four declared structures

Definition. An experiment consists of the following data.

• **External geometry.** The fixed configuration

$$(\overline{M}, M, g, N = 1, \mathscr{V}, U, \chi, c).$$

• **Names and locations.** At each semantic cut there is a finite set of live quantum names. Their Hilbert spaces are finite tensor products of qubits. A location map assigns each accessible port to a point of M. Several names may be co-located.

• **Accessible algebras.** For a declared local collection A of live names, the accessible algebra is the full matrix algebra on those names, tensored with the identity on other live names. A smaller algebra may be declared, but no subsequent operation may silently use an observable outside it.

• **Complete control and resources.** A finite instruction tree specifies allocations, permitted unitaries, destructive readouts, classical controls, input and output ports, schedules, location data, retained storage, and all auxiliary factors. Every classical control dependency is either preinstalled task data or a declared causal record wire. The background metric has no state-dependent update rule in this model.

A fixed classical history has one normalized pure state on its complete live quantum names. Unitary gates and pure allocations preserve this property. At a rank-one destructive readout, each supported outcome has a normalized pure residual. Density operators and recorded channels describe ensembles, inaccessible restrictions, and complete input–output comparisons; they do not assert that a classical mixture is one pure branch.

An untouched reference E may be included in a complete input. It has no available control or communication wire. Its storage is listed separately from the laboratory storage.

The clock instrument

Fix disjoint clock and matter names, with

$$H_C \cong \mathbb{C}^D, \qquad D = 2^q, \qquad q \geq 1,$$

and a finite-dimensional surviving space *H_X*. We consistently order the input factors as $C \otimes X$.

Choose an orthonormal clock basis

$$\{|\tau_k\rangle : 0 \leq k < D\}, \qquad P_k = |\tau_k\rangle\langle\tau_k|,$$

and the unitary

$$B_C|\tau_k\rangle = |k\rangle.$$

The program applies *B_C*, then computational-basis Enter to all clock names. It returns the *q*-bit code *k*, not the names *C*.

Define

$$D_k = \langle\tau_k|_C \otimes I_X : H_C \otimes H_X \to H_X.$$

The record has diagonal observable algebra

$$\mathscr{A}_R = \left\{ \sum_{k=0}^{D-1} a_k|k\rangle\langle k|_R : a_k \in \mathbb{C} \right\}.$$

Theorem. The specified readout has channel

$$\begin{aligned}\mathscr{M}_C(Z) &= \sum_{k=0}^{D-1} |k\rangle\langle k|_R \otimes D_k Z D_k^\dagger \\ &= \sum_{k=0}^{D-1} |k\rangle\langle k|_R \otimes \mathrm{Tr}_C\,[(P_k \otimes I_X)Z(P_k \otimes I_X)].\end{aligned}$$

It is completely positive and trace preserving. Its *k*-th subnormalized matter block is

$$\sigma_k = D_k \rho D_k^\dagger, \qquad p_k = \mathrm{Tr}\,\sigma_k.$$

For *p_k > 0*, the conditional matter state is *σ_k/p_k*. No normalized state is assigned to a zero-probability outcome. Every identity holds after tensoring with *id_E*.

Proof. After applying *B_C*, the destructive measurement bra for outcome *k* is

$$(\langle k\mathrm{rvert}\mathrm{B}_C) \otimes I_X = D_k.$$

The Kraus operator including the classical output is

$$L_k = |k\rangle_R \otimes D_k.$$

Consequently the channel is , and

$$\sum_k L_k^\dagger L_k = \sum_k P_k \otimes I_X = I_{CX}.$$

This proves complete positivity and trace preservation. The partial-trace expression follows by expanding in the clock basis. The block probabilities and conditional states are the specified Born semantics. Tensoring each operator identity with *I_E* proves the reference versions.

Theorem is the finite interface used in En09-A; its conditioning block is not, merely by its form, a metric or a stress tensor [CheungEn09].

Three interfaces, not one

For comparison, define the coherent recording isometry

$$W_{\mathrm{rec}} = \sum_k P_k \otimes |k\rangle_A,$$

where the output retains *C*, adds a quantum carrier *A*, and retains *X*. Define also the represented nondestructive instrument

$$\mathscr{I}^{\mathrm{nd}}(\rho) = \sum_k |k\rangle\langle k|_R \otimes (P_k \otimes I_X)\rho(P_k \otimes I_X).$$

Proposition. Among these three interfaces, only $\mathscr{M}_C$ removes C from the returned quantum context. Moreover,

$$(\mathrm{id}_R \otimes \mathrm{Tr}_C)\mathscr{I}^{\mathrm{nd}} = \mathscr{M}_C.$$

Coherent recording returns a quantum carrier and is not itself a classical recorded channel.

Proof. The assertions about returned names follow from the displayed domains and codomains. The partial-trace identity is Theorem . For a superposition of two clock basis states, coherent recording contains off-diagonal terms between the corresponding $C \otimes A$ states. Hence it is not generally block diagonal in A.

The equality after discarding C does not identify the larger nondestructive interface with destructive Enter [CheungI,CheungEn03].

The dictionary and its calibration limits

Time labels and reading events

Fix distinct ordered real numbers

$$t_0 < t_1 < \cdots < t_{D-1}.$$

The stored object is k. Its interpretation as t_k is classical metadata. The finite operator

$$\widehat{\mathrm{T}}_C = \sum_k t_k P_k$$

is a pointer observable, not the continuum coordinate function T.

There are two different admissible calibrations.

Definition.

• **Event-time calibration.** For a reading at the fixed event e, an input is exactly calibrated when

$$p_k > 0 \quad \Rightarrow \quad t_k = T(e).$$

Thus the returned label is the coordinate of the reading event.

• **History-label calibration.** Choose a finite event map

$$\kappa : \{0, \ldots, D-1\} \to M, \qquad T(\kappa(k)) = t_k.$$

A readout at an independently specified event e_r returns the label of $\kappa(k)$. It need not return $T(e_r)$. In the examples below all $\kappa(k)$ are earlier points on the sender's static worldline.

Proposition. Under distinct-label calibration, holds if and only if there is a unique j with $t_j = T(e)$ and

$$\rho = (P_j \otimes I_X)\rho(P_j \otimes I_X).$$

The readout then returns j with probability one. If $T(e) \notin \{t_k\}$, no normalized exactly calibrated input exists.

Proof. Distinctness permits at most one index j satisfying $t_j = T(e)$. Calibration and normalization give $p_j = 1$. Put $Q = I - P_j \otimes I_X$. Positivity and $Tr\ (Q\rho) = 0$ imply

$$0 = \mathrm{Tr}\,(\rho^{1/2} Q \rho^{1/2}) = \|Q\rho^{1/2}\|_{\mathrm{HS}}^2.$$

Therefore $Q\rho = \rho Q = 0$, proving the support statement. The converse follows immediately from .

Remark. A coherent history with several supported readings cannot make the single classical number $T(e_r)$ equal several distinct t_k. Calling t_k a geometric label is legitimate after κ is supplied. Calling every such outcome the actual time of one fixed event would be false. Random classical selection among different reading events is another valid construction, but the event selection must then be included as classical input data.

The three dictionary maps

Definition. The dictionary is the following explicit collection of maps.

• **Clock map.** The basis calibration B_C, the finite map $k \mapsto t_k$, and either the event support condition or the history-event map κ.

• **Rod map.** At each cut, a surviving carrier name a has a prescribed spatial location $x(a) \in U$, with rod label $\chi(x(a))$. In polar notation the corresponding local labels are $(r, \alpha\varphi, z)$.

• **Causal map.** The complete scheduled port graph is mapped to M so that every directed dependency edge maps to a future-causal pair. Ancestors of an output therefore map into its metric past. These maps are model data. They are not inferred from output probabilities alone.

A pre-read and post-read germ $s_{*}^{-}\prec s_{*}^{+}$ describes semantic order at one reading event. It need not denote two distinct geometric times. In particular, strict semantic sequencing does not imply a positive proper-time duration for an idealized local instruction.

The five time-related objects

The distinctions in XIX-A motivate the following ledger [CheungXIX].

1.13 toprule Object Status in this construction

midrule Finite-time harmonic relational scalar A separate classical prescription; not used to determine the conical lapse.

Inflaton clock and original slice-harmonic rods An optional cosmological attachment, conditional on full input geometry; Appendix .

Conical incidence parameter Metadata of a prescribed slicing family, not the finite clock factor or an automatically normalized time coordinate.

Density ramp A prescribed regulator, not an independently evolving clock degree of freedom.

Finite Enter clock C A live quantum factor, with a finite reading basis and an explicit calibration to geometric labels.

bottomrule

Lemma. In the present model class:

• even a harmonic scalar profile does not determine the lapse;

• a finite pointer is not the all-real coordinate T;

• an externally specified density ramp does not, by its specification alone, supply an independent dynamical clock;

• a slicing-family parameter is not identified with T without an additional calibration.

Proof. For (i), take any two positive constant lapses $N = a$ and $N = b$. The scalar $\xi = T$ is harmonic for both metrics: the coefficients and volume density are independent of T, so $\Box_{g_N} T = 0$. The scalar profile is identical, but

$$-g_N^{-1}(\mathrm{d}T, \mathrm{d}T) = \frac{1}{c^2 N^2}$$

depends on the lapse.

For (ii), the pointer has at most D distinct eigenvalues, whereas T takes every real value on M.

For (iii), a fixed regulator $r_{\varepsilon}(T) = r(T/\varepsilon)$ adds a prescribed function of external labels, not a new state variable or evolution equation. It can be used in a calibration if further data are supplied; that does not make an independent dynamical degree of freedom appear.

For (iv), a family of slices may be reindexed by any bijection of its parameter interval without changing the collection of slices. Neither the background metric nor the finite clock instrument selects one such indexing.

The original-versus-modified rod distinction is stronger than an indexing issue: Appendix gives a compact perturbative witness separating their quadratic lapses.

Nonuniqueness of the dictionary

Proposition. Finite record laws and complete matter channels in this construction do not determine the rod frame, the lapse, or the global deficit.

Proof. First, apply a spatial Euclidean isometry to the chosen developing chart and all prescribed carrier locations. The abstract name permutations and channels are unchanged, while their numerical rod coordinates change.

Second, retain the same event labels, sites, and scheduled channels, but compare the constant lapses $N = 1$ and $N = 2$. Every edge satisfying $\|\Delta x\| \le c\Delta T$ is causal for both. The same matrices produce exactly the same complete channels and record probabilities. Nevertheless, a static interval of coordinate duration ΔT has proper duration ΔT in the first calibration and $2\Delta T$ in the second.

Third, any two deficits in $(0, 2\pi)$ admit regular developing patches containing isometric sufficiently small Euclidean balls. Install the same finite experiment in such balls. The complete channels coincide, although the global circumference-to-radius ratios and conical gluing angles differ.

This is nonuniqueness relative to the supplied finite operational data. It does not assert that all the displayed geometric configurations are inequivalent under every possible change of coordinates.

Signature is an additional choice

Proposition. For $n \ge 2$, no rule assigns to each positive-definite inner product on an n-dimensional real vector space a Lorentzian bilinear form on that same space, using only that inner product and equivariantly under all linear changes of coordinates.

Proof. At the Euclidean inner product I, equivariance makes the assigned form invariant under every orthogonal transformation. Coordinate reflections force its off-diagonal entries to vanish. Coordinate permutations force its diagonal entries to coincide. It is therefore λI, which is definite or degenerate, not Lorentzian.

This reproduces the elementary signature refusal recorded in Oai365-A [CheungOai365]. Here we add a time factor, a negative sign, and a time orientation explicitly. A velocity diamond or polytope is not a substitute for that choice.

What destructive clock reading does and does not do

Consumption, nonreturn, and the surviving record

Proposition. For $D \ge 2$, $\mathscr{M}_C$ has no channel left inverse on its full $C \otimes X$ input space.

Proof. Let

$$|\chi_{\pm}\rangle = \frac{|\tau_0\rangle \pm |\tau_1\rangle}{\sqrt{2}},$$

and fix any matter state η_X. The distinct inputs

$$|\chi_{+}\rangle\langle\chi_{+}| \otimes \eta_X, \qquad |\chi_{-}\rangle\langle\chi_{-}| \otimes \eta_X$$

both have output

$$\frac{1}{2}(|0\rangle\langle 0|_R + |1\rangle\langle 1|_R) \otimes \eta_X.$$

A left inverse would send one output to two different inputs.

Remark. The proposition concerns the full clock–matter input space. It does not forbid preparing a fresh clock in a known basis state from a classical code. Indeed, on the block-diagonal class

$$\rho = \sum_k P_k \otimes \sigma_k,$$

a channel can reconstruct the displayed density operator by preparing $|\tau_k\rangle$ conditional on k. This is recovery of a classical sector, not recovery of arbitrary clock coherence and not return of the consumed names.

Proposition. Identical clock record laws do not determine the surviving matter state, even on exactly event-calibrated inputs.

Proof. Choose orthogonal matter vectors $|a\rangle$, $|b\rangle$. Both inputs

$$P_0 \otimes |a\rangle\langle a|, \qquad P_0 \otimes |b\rangle\langle b|$$

return the deterministic record 0. Their matter residuals are orthogonal. More generally, postcomposing every selective block with either of two different trace-preserving matter channels preserves the clock probabilities while changing the residual instrument.

Record compatibility and residual compatibility are therefore different assertions, as emphasized in I, VIII-A, and

En09-A [CheungI,CheungVIIIA,CheungEn09].

Proposition. For every input operator,

$$\mathrm{Tr}_R \mathscr{M}_C(Z) = \mathrm{Tr}_C Z.$$

This identity is reference preserving.

Proof. Writing $Z = \sum_{j,k} |\tau_j\rangle\langle\tau_k| \otimes Z_{jk}$, both sides equal $\sum_k Z_{kk}$. The same computation retains any reference factor.

The geometric coordinate has not disappeared: for example, an unchanged classical record may remain stored on a static worldline while T increases along that worldline. A fixed record of an earlier reading is not a replacement for all later external time.

No finite canonical commutator

Proposition. For nonzero finite-dimensional H_C and $\hbar > 0$, no matrices A, H satisfy

$$[A, H] = i\hbar I_C.$$

Proof. The trace of a matrix commutator is zero. The trace of the right side is $i\hbar D \neq 0$.

Thus T_C is not an exact canonical time operator conjugate to a finite clock Hamiltonian. The continuum coordinate T remains a classical function on the supplied manifold.

Basis change is not generally coordinate change

Suppose a unitary U_C is applied before the calibrated readout. Its effective pointer observable on the previous input is

$$U_C^\dagger \widehat{\mathrm{T}}_C U_C.$$

Proposition. Assume the t_k are distinct. If

$$U_C^\dagger \widehat{\mathrm{T}}_C U_C = f(\widehat{\mathrm{T}}_C)$$

for a function f on the finite spectrum, then U_C permutes the one-dimensional eigenspaces of T_C, up to phases. Consequently, a general unitary basis change is not even a finite relabeling of the original pointer, much less a spacetime coordinate transformation.

Proof. The conjugated operator has simple spectrum. Hence $f(T_C)$ must also have distinct eigenvalues. Its eigenvectors are the original $|\tau_k\rangle$, whereas those of the conjugated operator are $U_C^\dagger|\tau_k\rangle$. Simplicity identifies these two collections of one-dimensional eigenspaces by a permutation.

For a concrete failure, take $D = 2$, use the computational reading basis, and apply Hadamard to the input $|0\rangle$. The originally deterministic reading becomes a two-outcome distribution. A deterministic relabelling of the original reading cannot do this.

Even an allowed finite relabelling requires more to define a smooth, time-oriented coordinate transformation. If an actual coordinate change $T' = f(T)$, with $f' > 0$, is supplied, tensor transformation gives in the zero-shift case

$$-c^2 N(T,x)^2 \, \mathrm{d}T^2 = -c^2 \left(\frac{N(f^{-1}(T'),x)}{\dot{f}(f^{-1}(T'))} \right)^2 \mathrm{d}T'^2.$$

No such transformation law follows from applying U_C alone. A simultaneous passive transformation of states and observable matrices is instead a change of representation.

A nontrivial finite history readout

The Page–Wootters literature motivates distinguishing extraction of conditional blocks from additional dynamical hypotheses [PageWootters,CheungEn09]. The following finite example proves exact conditioning without claiming a stationary-universe construction.

Choose any finite-dimensional H_X, a self-adjoint H_X^{gen}, and

$$U_k = \exp\left(-i t_k H_X^{\mathrm{gen}}/\hbar\right).$$

Define the isometry

$$J : H_X \to H_C \otimes H_X, \qquad J|\psi\rangle = \frac{1}{\sqrt{D}} \sum_{k=0}^{D-1} |\tau_k\rangle \otimes U_k|\psi\rangle.$$

Proposition. For every operator Z on H_X,

$$\mathscr{M}_C(JZJ^\dagger) = \frac{1}{D} \sum_{k=0}^{D-1} |k\rangle\langle k|_R \otimes U_k Z U_k^\dagger.$$

This equality holds with every untouched reference. A record-controlled application of $U_k^\dagger$ returns the original matter input, including its reference correlations. It does not invert $\mathscr{M}_C$ on the full clock–matter input space.

Proof. Orthogonality gives

$$J^\dagger J = \frac{1}{D} \sum_k U_k^\dagger U_k = I_X, \qquad D_k J = D^{-1/2} U_k.$$

Substitution into proves . Controlled inversion gives $D^{-1} \sum_k |k\rangle\langle k| \otimes Z$. These are operator identities and remain true with a reference. The inverse acts only on the encoded matter task; the phase-erasure counterexample of Proposition still applies to the full input space.

For a concrete preparation, allocate the clock in $|\tau_0\rangle$, rotate it to $D^{-1/2} \sum_k |\tau_k\rangle$, and apply the controlled unitary $\sum_k P_k \otimes U_k$. All these operations can be declared at the sender's location before the reading.

Choose a static sender $x_0 \in U$, set

$$\kappa(k) = (t_k, x_0), \qquad T(e_r) > \max_k t_k,$$

and use history-label calibration. The labels refer to the declared points $\kappa(k)$, not to different actual locations of e_r . The displayed preparation does not, by itself, prove that a physical matter trajectory visited all the conditional states at those earlier events. It constructs a calibrated finite history encoding.

Fresh allocation and thermodynamic boundaries

After consumption, a later reading may allocate a new factor C'. For example,

$$\eta_{RX} \longmapsto \eta_{RX} \otimes |0\rangle\langle 0|_{C'}$$

preserves the existing record and supplies a new blank. It neither returns the old name nor recovers an unknown phase lost by Enter.

A channel specification does not determine a heat or work bill [CheungEn07]. For instance, resetting a memory by swapping with a blank ancilla has the same reduced reset channel for every common energy gap $\varepsilon > 0$. With

$$H_M = H_A = \varepsilon|1\rangle\langle 1|,$$

an input excited-state probability p produces energy transfers

$$\Delta E_M = -\varepsilon p, \qquad \Delta E_A = \varepsilon p.$$

The blank is consumed and the original information moves to the ancilla. Restoring that ancilla, its preparation resources, and all controls is a separate cyclic-restoration task.

Schedule locality and the metric causal dictionary

A finite local execution model

Let Λ be a finite set of static sites in U, with finite-dimensional site spaces H_a. Fix times

$$T_m = T_{\text{in}} + m\delta, \qquad \delta > 0.$$

At each tick m, a partition $\mathscr{P}_m$ of Λ is supplied. Each block obeys

$$\|\chi(x_a) - \chi(x_b)\|_2 \le c\delta \quad (a, b \in B,\ B \in \mathscr{P}_m).$$

A tick is one complete product channel

$$\Phi_m = \bigotimes_{B \in \mathscr{P}_m} \Phi_{m,B}.$$

All quantum ports, records, controller memories, and resource wires that are available during execution must be included in the finite network. An external reference has no execution wire.

The access contract is stroboscopic: input and output access is specified at tick boundaries. No continuous-time Hamiltonian or within-tick apparatus dynamics is asserted. A two-site gate is not treated as an instantaneous equal-time interaction. Its dependencies run from input ports at *T_m* to output ports at *T_{m + 1}*.

For $Y \subseteq \Lambda$, write

$$\mathcal{A}(Y) = \mathcal{B}(\text{bigotimes}_{a \in Y} H_a) \otimes I_{\Lambda \setminus Y}.$$

For a terminal cut *n*, define backward sets by

$$N_n(Y) = Y, \qquad N_m(Y) = \bigcup \{B \in \mathcal{P}_m : B \cap N_{m+1}(Y) \neq \varnothing\}.$$

Theorem. For the complete scheduled channels,

$$\Phi_0^* \cdots \Phi_{n-1}^*(\mathcal{A}(Y)) \subseteq \mathcal{A}(N_0(Y)).$$

The terminal reduction to *Y* factors through the initial reduction to *N_0(Y)*. The factorization is reference preserving. Changing a complete gate outside the ancestor subnetwork does not change the terminal reduction, when wiring and included gates are held fixed.

Proof. At one backward step, an output observable is the identity on every block disjoint from its current support. The adjoint of a trace-preserving channel is unital, so such a block remains an identity. Other blocks can enlarge support only to their union. Induction gives .

To obtain a reduced channel, append any fixed state on the initial complement of *N_0(Y)*, apply the complete evolution, and take the output reduction to *Y*. The adjoint inclusion shows that this result is independent of the appended state and agrees with the actual output for every input, including correlated inputs. The same adjoint argument tensored with *id_E* proves the reference version.

On an excluded gate all pulled-back observables are identities. Any replacement complete channel preserves those identities.

This is the local proof underlying the prescribed-network comparisons in En03-A, En05-A, and En06-A [CheungEn03,CheungEn05A,CheungEn06A]. It concerns complete unconditioned channels with records retained, not normalized postselection.

Theorem. Every directed scheduled dependency path in this execution model maps to a future-causal curve in *M*. In particular,

$$a \in N_m(\{b\}) \quad \Rightarrow \quad (T_m, x_a) \in J_g^-((T_n, x_b)).$$

Operational influence under trace-preserving local interventions is contained in the scheduled dependency relation and hence in the prescribed metric causal relation.

Proof. Every elementary dependency connects two sites in one permitted block across duration δ. Condition and Lemma supply a future-causal segment for that pair. Concatenation supplies a causal curve along a dependency path.

If an intervention acts outside the ancestors, its adjoint acts on an identity in the backward calculation, and therefore cannot change an unconditioned terminal expectation. This proves the influence inclusion.

Remark. Declare a two-site scheduled block but choose its channel to be the identity. Its formal block ancestry includes both input sites for each output, although no cross-site influence occurs. Thus the scheduled region can strictly exceed the actual influence region. An entangled input does not invalidate the theorem, but comparing only individual marginals is insufficient for a claim involving joint outputs and references.

Scheduler consistency does not select the Lorentz cone

Proposition. Acyclic dependency, finite local updates, and deterministic schedule-independent records do not imply a Lorentzian generated cone.

Proof. Take events indexed by $\mathbb{N}^4$, with directed edges $n \to n + e_i$, $i = 1, \ldots, 4$. Use a deterministic bit relay on any finite predecessor-closed subnetwork: the recorded value is the Boolean OR of incoming values and a local

input bit. Every update occurs after its predecessors. The graph is acyclic, has bounded degree, and every finite experiment terminates. Associativity and determinism make records independent of the legal interleaving. A source bit can be detected at every reachable descendant by setting other inputs to zero.

The closed cone generated by edge displacements is

$$K = \mathbb{R}^4_{\geq 0}.$$

It is not linearly isomorphic to the four-dimensional Lorentz cone. Indeed, K has four extreme rays, whereas the Lorentz cone has infinitely many. An invertible linear map preserves extreme rays.

This is a local counterexample to the proposed inference, not a reproduction of every additional clause of En08-A's stronger countermodel [CheungEn08A].

There is a relevant positive classification: a proper, full-dimensional, homogeneous, strictly convex cone in dimension at least three is linearly isomorphic to a Lorentz cone [ItoLourenco]. Here ``proper'' means closed, convex, pointed, and with nonempty interior; strict convexity is understood in the cone sense. This theorem classifies a cone satisfying its additional hypotheses; it does not construct those hypotheses from a scheduler.

XXII-F records the additional extraction, invariance, approximation, resource, and layer-separation obligations for an operational instantiation [CheungXXIIF]. We do not claim to discharge that existence target. Declaring is not a metric-independent operational extraction rule.

One finite complete-channel null transport protocol

The target-adapted contract

Fix *x_0*, *x_1* ∈ *U*, *x_0* ≠ *x_1*, with

$$L = \|\chi(x_1) - \chi(x_0)\|_2.$$

Fix a departure time *T_d*. The prescribed target pair is

$$p = (T_{\mathrm{d}}, x_0), \qquad q = (T_{\mathrm{d}} + L/c, x_1).$$

The developed straight segment is future null.

Choose any integer $m \geq 1$. Install the $m + 1$ sites

$$\chi(x_j) = \chi(x_0) + \frac{j}{m}(\chi(x_1) - \chi(x_0)), \qquad 0 \leq j \leq m,$$

where the notation *x_m = x_1* is understood. Each site has a register *A_j* ≅ ℂ^*d*, with *d = 2^p* ≥ *2*. Set

$$a = \frac{L}{m}, \qquad \delta = \frac{L}{mc}.$$

At tick *j = 1, …, m*, apply SWAP to *A_{j − 1}* ⊗ *A_j* and identities to all other sites.

The station locations and tick duration are task data. They are not selected by the unknown payload or inferred from its entanglement.

Theorem. Prepare *A_1, …, A_m* in the fixed blank $\sigma = |0\rangle\langle 0|$, independently of the payload and any reference. For every state *ρ_A_0E*, after tick *j* the state is

$$\rho_{A_jE} \otimes \mathrm{bigotimes}_{\mathrm{substack}\, 0 \leq i \leq m}$$

under the fixed register identifications. Consequently:

• the logical source-to-destination channel is exactly *id_d*, including every untouched reference;

• all non-output route registers end in a jointly payload-independent blank state;

• the arrival event is exactly *q*;

• every scheduled dependency is metric causal;

• the protocol uses *m* two-register SWAP gates, *m + 1* payload-sized registers, and *m* ticks.

Proof. At *j = 0*, is the preparation. At tick *j*, register *A_j* is blank and independent. SWAP exchanges it with the entire current payload factor *A_{j − 1}*, including all correlations with *E*. The previous site becomes blank,

proving the invariant by induction.

At $j = m$, the destination-reference state is the original payload state under the fixed identification. The remaining registers are blank, proving the complete-channel assertions.

Each active pair has separation $a = c\delta$. Hence its input-output dependencies satisfy the local causal certificate. The payload positions at successive ticks trace the prescribed null segment. Finally $m\delta = L/c$, and the gate and register counts follow from the explicit schedule.

Remark. This theorem uses a target-adapted substrate. It is not a claim that an arbitrary null displacement is exactly attained on one fixed lattice. En08-B instead supplies resource-controlled approximation for a prescribed null boundary on its declared lattice family [CheungEn08B]. Our construction has the same essential reference-preserving permutation identity, but a different spatial permission contract. No resource formula is transferred between these models.

For increasing m, the gate radius and tick duration tend to zero, while total travel time remains L/c and register count grows. This family is not a continuum quantum field theory.

Remark. The theorem certifies the complete finite channel and the causal embedding of its tick-boundary dependencies. It supplies no continuous-time Hamiltonian that implements a two-site SWAP inside one tick, no detector model for its intermediate dynamics, and no field-theoretic derivation of the channel. Those are additional physical-realization obligations, not consequences of the endpoint permutation.

Composition with the clock instrument

Let the surviving clock-conditioned matter be the register A_0. Write $\mathscr{T}_m$ for the complete route channel with its blank preparation, retaining all route outputs. Let ι identify A_0 with A_m.

Corollary. For every input state on $C \otimes A_0 \otimes E$, clock readout at the sender followed by the route has complete output

$$\sum_k |k\rangle\langle k|_{R_{\mathrm{sender}}} \otimes (\iota \otimes \mathrm{id}_E)(\sigma_{A_0E,k}) \otimes \mathrm{bigotimes}_{i=0}^{m-1} \sigma_{A_i},$$

where

$$\sigma_{A_0E,k} = (D_k \otimes I_E)\rho(D_k^\dagger \otimes I_E).$$

The clock factor C is absent.

Proof. Apply Theorem to each subnormalized positive block of Theorem , and sum the blocks. Linearity gives .

The record in remains at the sender. Writing a joint output state is not instantaneous delivery of that record to the receiver. If a receiver operation is to depend on k, a causal classical record wire must be added, or the record-controlled operation must be performed locally before departure.

Resource and robustness ledgers

With a q-qubit clock and p-qubit payload registers, the explicit laboratory quantum peak is at most

$$Q_{\mathrm{lab}} = q + p(m + 1).$$

After clock consumption it is $p(m + 1)$ until the route residuals are released. The clock record has q classical bits. An untouched r-qubit reference contributes r to a separate external-storage ledger, or to a total ledger if that convention is chosen. The preinstalled schedule, site metadata, and blank preparations are declared resources, not outputs supplied by the record.

The peak in does not include undeclared synthesis ancillas: the named local unitaries are permitted primitives of this model. No compilation into a smaller universal gate library is asserted.

Proposition. For compatible channels define

$$d_\diamond(\Phi, \Psi) = \frac{1}{2}\|\Phi - \Psi\|_\diamond.$$

If the implemented preparation, readout, m route gates, and output interface differ from their ideal versions by at most ε_prep, ε_read, ε_1, …, ε_m, and ε_out, respectively, then the complete output channel differs by at most

$$\varepsilon_{\mathrm{prep}} + \varepsilon_{\mathrm{read}} + \sum_{j=1}^{m} \varepsilon_j + \varepsilon_{\mathrm{out}}.$$

Proof. Insert and subtract hybrid compositions in which one component at a time is replaced by its ideal version. The triangle inequality and contraction of the diamond norm under pre- and postcomposition by channels give the bound.

This estimate is not exact causal containment of an erroneous device. Nor does a per-gate error tending to zero imply a vanishing total error for an arbitrarily growing number of gates.

Dynamics without an invented source

The Einstein comparator

For a declared smooth geometric metric q, a declared covariant source tensor $T_{\mu\nu}$, constants Λ and $\kappa_{\mathrm{grav}} \neq 0$, define the comparison equation

$$G_{\mu\nu}(q) + \Lambda q_{\mu\nu} = \kappa_{\mathrm{grav}} \mathrm{T}_{\mu\nu}.$$

This equation is not a rule already contained in . A finite conditional density matrix is not a covariant spacetime tensor until a field, observable, state, renormalization prescription, and map to tensor-valued distributions are supplied.

Proposition. A symmetric, conserved, traceless tensor on a background need not source that background through .

Proof. In a flat four-dimensional chart with metric $\eta = \mathrm{diag}(-1, 1, 1, 1)$, let

$$\mathrm{t}_{\mu\nu} = \mathrm{diag}\,(3e, e, e, e), \qquad e > 0.$$

It is constant and symmetric, its covariant divergence vanishes, and $\eta^{\mu\nu} t_{\mu\nu} = 0$. For $\Lambda = 0$, the left side of is zero while the right side with $T = t$ is nonzero.

This is the four-dimensional version of the elementary refusal in VIII-C; local Ward identities likewise do not, by themselves, supply Einstein dynamics [CheungVIIIC,CheungWard].

Proposition. If the components of a nondegenerate metric are constant in all spacetime coordinates, its connection coefficients and curvature tensors vanish. Positive constant rescaling preserves this conclusion. A family of such constant metrics has zero linearized Einstein tensor for its parameter variation.

Proof. Every spacetime derivative of every component vanishes, so

$$\Gamma^{\rho}_{\mu\nu} = \frac{1}{2} q^{\rho\sigma} (\partial_\mu q_{\sigma\nu} + \partial_\nu q_{\sigma\mu} - \partial_\sigma q_{\mu\nu}) = 0.$$

The curvature formulas then vanish. Constant rescaling and differentiation with respect to an external family parameter retain spacetime-constant components.

The needed statement is the spacetime-constant one [CheungXXIIE]. The phrase ``spatially constant" alone would be insufficient: in units $c = 1$,

$$q = -\,\mathrm{d}T^2 + e^{2HT}(\,\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2), \qquad H \neq 0,$$

is spatially homogeneous but has

$$R_{00} = -3H^2, \qquad R_{ij} = 3H^2 q_{ij}, \qquad R = 12H^2.$$

These follow directly from $\Gamma_{ij}^{0} = H q_{ij}$ and $\Gamma_{0j}^{i} = H \delta_j^i$.

Application to the constructed background

By Proposition , the main conical metric has $G_{\mu\nu} = 0$ on its regular part. This says nothing about a distributional defect source on $\mathscr{W}$, which is not constructed here. Its conical gluing is prescribed rather than sourced by the Enter residual.

More generally, if a cosmological Einstein–scalar background is adjoined, its equations must be imposed on the geometric side. A quotient or relational scalar construction does not insert a missing metric source equation. This

is the equation-space discipline illustrated by III and VI [CheungIII,CheungVI].

Remark. The experiment has no map

$$\sigma_k \text{longmapsto } g_k \quad \text{or} \quad \sigma_k \text{longmapsto } \mathrm{T}_{\mu\nu,k}$$

in its specification. Consequently conditioning, changing a reading basis, forgetting a record, or allocating a fresh clock does not change the prescribed metric. Any such update would be an extension of the model with its own consistency and dynamics obligations.

Existence and exact mathematical release

Theorem. Fix $0 < \mu < 2\pi$, $c > 0$, a regular convex developing patch U, a clock dimension $D = 2\hat{}q \geq 2$, and a payload dimension $d = 2\hat{}p \geq 2$. Then the following finite operational package exists.

• Its smooth background is the time-oriented $3 + 1$-dimensional Lorentzian manifold

$$M = \mathbb{R}_T \times C_\mu^\circ \times \mathbb{R}_z, \qquad g = -c^2\,\mathrm{d}T^2 + \mathrm{d}r^2 + \alpha^2 r^2\,\mathrm{d}\varphi^2 + \mathrm{d}z^2.$$

Its completion has the declared conical worldsheet $\mathscr{W}$.

• It has one designated live clock factor C, finite matter names, a location map, accessible algebras, and a complete finite control and resource interface.

• Destructive clock Enter is exactly , including every untouched reference. It consumes C, writes a classical code, and retains the subnormalized conditional matter block.

• An explicit dictionary matches the code to geometric time labels and the surviving carrier names to three spatial rods. Exact equality with the actual reading-event coordinate holds on the event-calibrated support class of Proposition . Nontrivial multi-outcome history readout instead uses the separate event map κ.

• Scheduled dependencies are contained in the prescribed metric causal relation. Actual intervention influence is contained in scheduled dependency.

• Every prescribed straight future-null displacement inside the patch has the finite target-adapted SWAP protocol of Theorem , with exact complete-channel correctness and the resource ledger . Its composition with clock readout is .

• The clock instrument has no universal left inverse on its full input space, and no finite canonical time commutator is available. The supplied metric and its time coordinate remain after clock consumption. Signature, spatial chart, speed, lapse, calibration, and transport permissions are inputs. Einstein dynamics are not derived.

Proof. The geometric assertions are Propositions and . Choose any finite set of sites in U, assign qubit registers, and prescribe the finite primitives of Definition ; this supplies item 2.

Theorem proves item 3. For item 4, choose distinct t_k and a static sender x_0. The family

$$e_k = (t_k, x_0), \qquad \rho_k = P_k \otimes \rho_X$$

is a nonempty event-calibrated family. A classical task input selects which e_k and preparation are used. For a pure matter input these are pure complete laboratory preparations. Thus actual event-time matching is exhibited, not merely postulated.

For a separate nontrivial history experiment, choose $\kappa(k) = (t_k, x_0)$, read at a later fixed event, and use the finite preparation . Proposition gives all D outcomes and their complete conditional outputs. The rod map is the explicit developing chart χ.

Theorems and give item 5. The sites , their partition schedule, Theorem , and Corollary give item 6. All preparations, clocks, site factors, records, and references have the declared ledger.

Propositions and prove item 7. The geometry is fixed metadata throughout the specification, so no instrument operation removes or changes its coordinate function. No source-extraction or metric-update map has been introduced.

Corollary. In this model, Enter acts on a finite quantum encoding calibrated to the metric's time axis. It can change the selected clock observable through preceding local gates, extract a time label, condition surviving

matter, and enable declared causal classical control. It does not consume a spacetime dimension, move the readout event backward in *T*, or derive a new metric.

Proof. The positive statements are the specified unitary and instrument operations. Their outputs and locations are those already proved. The negative statements follow because *M*, *g*, *T* are unchanged classical components of the specification.

What remains outside the release

The following six stronger implications have not been supplied.

• A chart reconstructed from records without a predeclared location map.

• Lapse coefficients reconstructed from records without a geometric clock and background prescription.

• A Lorentz cone selected by scheduler consistency alone. The conditional classification and XXII-F's operational existence target remain distinct from our prescribed cone.

• A covariant stress tensor extracted from the residual that sources the same metric through a proved dynamics law.

• The native quantum-lapse construction and its compatible joint regulator removal.

• A calibration to proper time derived from records rather than supplied and checked against a declared metric and observer.

The last item does not retract Proposition : for the chosen unit-lapse static observers, equality of coordinate and proper-time increments is proved. What is not derived is the calibration of a physical device to that ideal geometric clock.

Release. A prescribed spacetime and a finite Enter experiment have been paired by written maps. This is a spacetime model under Enter in the stated operational sense. It is not spacetime emergence, holographic reconstruction, a quantum field theory, or a derivation of gravity. No operational wormhole certificate is identified with a geometric bridge. No catalogue family is a premise.

appendix

Optional original-rod lapse attachment

This appendix gives a self-contained classical composition interface. It does not replace the conical metric by a polynomial lapse or claim to reproduce every native input of XX-A or Em02-E. Those manuscripts supply their own frozen background, clock, rods, and quantum admission contracts [CheungXXA,CheungEm02E].

Throughout the appendix set $c = 1$, and use ε for a perturbation parameter. It is neither a schedule tick nor a clock reading.

The full geometric input

Suppose a smooth Lorentzian metric q_{ε} and a scalar clock $\mathscr{T}_{\varepsilon}$ are supplied on a region where $d\mathscr{T}_{\varepsilon}$ is timelike. Define the positive raw lapse

$$N_\varepsilon = \left(-q_\varepsilon^{-1}(\mathrm{d}\mathscr{T}_\varepsilon, \mathrm{d}\mathscr{T}_\varepsilon)\right)^{-1/2}.$$

Supply original slice-harmonic rods and suppose the resulting coordinate map

$$\mathscr{C}_\varepsilon : x \mathrm{longmapsto} (\mathscr{T}_\varepsilon(x), Y^1_\varepsilon(x), Y^2_\varepsilon(x), Y^3_\varepsilon(x))$$

is locally invertible, with inverse α_{ε}. Then the relational lapse is

$$n_\varepsilon = N_\varepsilon \circ \alpha_\varepsilon.$$

In these relational coordinates the full metric has ADM form

$$q_\varepsilon = -n_\varepsilon^2\,\mathrm{d}\tau^2 + \gamma_{\varepsilon,ij}(\mathrm{d}y^i + \beta^i_\varepsilon\,\mathrm{d}\tau)(\mathrm{d}y^j + \beta^j_\varepsilon\,\mathrm{d}\tau).$$

Indeed, writing the metric as $A\,d\tau^2 + 2B_i\,d\tau\,dy^i + \gamma_{ij}\,dy^i\,dy^j$, set $\beta^i = \gamma^{ij}B_j$ and $n^2 = -A + \gamma^{ij}B_iB_j$; completing the square gives , and inversion gives $q^{\tau\tau} = -n^{-2}$.

Thus a lapse alone does not specify the metric. Along a curve of constant rods, proper-time rate is

$$\frac{d\tau_{\text{proper}}}{d\tau} = \sqrt{n_\varepsilon^2 - \gamma_{\varepsilon,ij}\beta_\varepsilon^i\beta_\varepsilon^j},$$

when the curve is timelike. Along a normal curve *dy^i = −β_{ε}^i dτ*, it is *n_{ε}*. Constant rods and normal observers must not be silently identified.

If *q_{ε}*, *φ_{ε}* are required to satisfy Einstein–scalar equations, those equations are additional geometric input. Merely supplying *n_{ε}* does not impose them.

The second-order composition formula

Work in background-adapted coordinates for which the background label map is the identity. Suppose the raw lapse has factorial jets

$$N_\varepsilon = N_0 + \varepsilon N_1 + \frac{\varepsilon^2}{2}N_2 \bmod \varepsilon^3, \qquad N_0 = N_0(t).$$

At fixed clock label, let the forward spatial rod map be

$$R_\varepsilon(\tau, x) = x + \varepsilon Y_1(\tau, x) + \frac{\varepsilon^2}{2}Y_2(\tau, x) \bmod \varepsilon^3.$$

Let the inverse time map at fixed spatial argument be

$$\theta_\varepsilon(\tau, x) = \tau + \varepsilon u_1(\tau, x) + \frac{\varepsilon^2}{2}u_2(\tau, x) \bmod \varepsilon^3.$$

The full inverse first inverts *R_{ε}*, then substitutes the resulting argument into *θ_{ε}*.

Lemma. Writing

$$\alpha_\varepsilon = \mathrm{id} + \varepsilon\alpha_1 + \frac{\varepsilon^2}{2}\alpha_2 \bmod \varepsilon^3,$$

one obtains

$$\alpha_1^0 = u_1, \qquad \alpha_1^i = -Y_1^i,$$

and

$$\alpha_2^0 = u_2 - 2Y_1^i\partial_i u_1, \qquad \alpha_2^i = 2Y_1^j\partial_j Y_1^i - Y_2^i.$$

Proof. Substitution into *R_{ε}(τ, S_{ε}(τ, y)) = y* gives

$$S_\varepsilon = y - \varepsilon Y_1 + \frac{\varepsilon^2}{2}(2Y_1^j\partial_j Y_1 - Y_2).$$

Expanding *θ_{ε}(τ, S_{ε}(τ, y))* gives the time component, including *−2Y_1^i∂_iu_1*.

Theorem. Under these hypotheses,

$$n_\varepsilon = N_0 + \varepsilon n_1 + \frac{\varepsilon^2}{2}n_2 \bmod \varepsilon^3,$$

where

$$n_1 = N_1 + \dot{N}_0 u_1,$$
$$n_2 = N_2 + 2u_1\dot{N}_1 + \dot{N}_0 u_2 + \ddot{N}_0 u_1^2 - 2Y_1^i\partial_i n_1.$$

The dot is differentiation in the background coordinate used in this expansion. The coefficient *Y_2* does not enter *n_2*.

Proof. Taylor composition gives

$$n_1 = N_1 + \alpha_1^\mu\partial_\mu N_0,$$
$$n_2 = N_2 + 2\alpha_1^\mu\partial_\mu N_1 + \alpha_2^\mu\partial_\mu N_0 + \alpha_1^\mu\alpha_1^\nu\partial_\mu\partial_\nu N_0.$$

Since *N_0* depends only on time, insertion of Lemma yields

$$n_2 = N_2 + 2u_1\dot{N}_1 + \dot{N}_0u_2 + \ddot{N}_0u_1^2 - 2Y_1^i\partial_i N_1 - 2\dot{N}_0Y_1^i\partial_i u_1.$$

The final two terms are *−2Y_1^i∂_i(N_1 + Ṅ_0u_1)*, giving . The coefficient containing *Y_2* multiplies *∂_iN_0 = 0*.

The shape agrees with the supplied classical formulas in XX-A and Em02-E. Equality of this algebraic shape is not identification of every clock normalization, background coordinate, raw coefficient, or boundary condition in those manuscripts.

A formal two-jet is also not, by itself, an exact positive lapse or an exact Einstein solution. When an exact smooth input family is supplied, positivity and local coordinate invertibility persist on a fixed compact comparison region for sufficiently small *ε*. When only formal data are supplied, the conclusion of Theorem remains a formal one.

The gauge composition identity

Proposition. Suppose a coordinate change *F* acts on the supplied objects so that

$$N_\varepsilon' = N_\varepsilon \circ F, \qquad \mathscr{C}_\varepsilon' = \mathscr{C}_\varepsilon \circ F.$$

Then

$$\alpha_\varepsilon' = F^{-1} \circ \alpha_\varepsilon, \qquad n_\varepsilon' = n_\varepsilon.$$

The same statement holds coefficientwise modulo *ε^3*.

Proof. Invert the second covariance identity, then compose:

$$N_\varepsilon' \circ \alpha_\varepsilon' = N_\varepsilon \circ F \circ F^{-1} \circ \alpha_\varepsilon = N_\varepsilon \circ \alpha_\varepsilon.$$

Formal composition gives the jet statement.

This proposition assumes the stated covariance of the actual clock and rod prescription, including its boundary data. It is not a quantum Ward identity and does not establish those covariance hypotheses for an unspecified prescription.

A compact rod-detection witness

The following local calculation reproduces the relevant distinction in XIX-A without using it as an unexamined premise [CheungXIX].

Let *a(t) > 0*, and assume *H = ȧ/a ≠ 0* on a comparison interval. Choose a nonconstant *ψ ∈ C_c^{∞}(ℝ^3; ℝ)* and a nonzero smooth time bump *b* supported strictly inside that interval. Set

$$A = b(t)\psi, \qquad D = -b(t)\Delta\psi,$$

and consider the off-shell metric jets

$$q_\varepsilon = -(1 + 2\varepsilon A)\,\mathrm{d}t^2 + a(t)^2(1 + 2\varepsilon D)\delta_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j \bmod \varepsilon^3, \qquad \mathscr{T}_\varepsilon = t.$$

Original rods are harmonic on the *t*-slices with asymptotic Cartesian boundary condition. Modified rods are harmonic on the slices of

$$\Theta_\varepsilon = t + \varepsilon D/H + O(\varepsilon^2),$$

with the same spatial boundary condition. The time label used in both relational lapse observables remains *t*.

Proposition. Let *Y_1* and *Z_1* be the first original and modified rod variations. Then

$$Y_1^i = b\,\partial_i\psi, \qquad Z_1^i = 0,$$

and the quadratic relational lapses satisfy

$$n_{2,Y} - n_{2,Z} = -2b(t)^2|\nabla\psi|^2.$$

A nonnegative compact spacetime test detects a strictly negative pairing.

Proof. For *γ_ij = a^2(1 + 2εD)δ_ij*, expansion of

$$\Delta_\gamma f = (\det\gamma)^{-1/2}\partial_j((\det\gamma)^{1/2}\gamma^{jk}\partial_k f)$$

gives $\Delta_{\gamma}x^i = \varepsilon a^{-2}\partial_i D + O(\varepsilon^2)$. Hence

$$\Delta Y_1^i = -\partial_i D = b\,\partial_i \Delta\psi,$$

whose decaying solution is $b\,\partial_i\psi$.

A modified slice through background time τ has $t = \tau - \varepsilon D/H + O(\varepsilon^2)$. The change in $a(t)^2$ cancels the first-order factor $2\varepsilon D$. Terms involving spatial derivatives of the embedding are second order. Thus its induced spatial metric is $a(\tau)^2\delta_{ij} + O(\varepsilon^2)$, and the decaying first harmonic correction is $Z_1^i = 0$.

The common raw lapse is

$$N_\varepsilon = \sqrt{1 + 2\varepsilon A} = 1 + \varepsilon A - \frac{\varepsilon^2}{2}A^2 \bmod \varepsilon^3.$$

The unchanged time coordinate gives $u_1 = u_2 = 0$, $n_1 = A = b\psi$, and the same raw N_2 for both prescriptions. Equation therefore gives

$$n_{2,Y} - n_{2,Z} = -2(Y_1^i - Z_1^i)\partial_i n_1 = -2b^2|\nabla\psi|^2.$$

Choose a nonnegative compact test supported where $b \neq 0$ and $\nabla\psi \neq 0$, positive on a nonempty open set. Its pairing is strictly negative.

The witness is off shell. It proves that the functional prescriptions differ; it is not a claim that every on-shell cosmological configuration distinguishes them.

Quantum admission is not supplied by the classical formula

Before a quantum version could be attached, one would need, at minimum:

• a normalized coupled free-field and gauge-fixed input with stated domains and propagators;

• an admissible state or covariance with positivity and the required singularity conditions;

• defined composite lapse and rod insertions, with the complete quadratic terms and a specified renormalization freedom;

• compatible gauge or Ward operations on those same objects, including the transformation of all required jets;

• uniform estimates controlling the nonlocal rod tails, ultraviolet operations, and the proposed regulator removals in one common output topology. This is an attachment checklist, not a claim to replace the full native admission contracts of XX-A or Em02-E. Their classical second-order release is not a quantum coefficient [CheungXXA,CheungEm02E].

The clock channel neither defines an absent field product nor proves any of these estimates. The characteristic-product programme at I3/J2 is a different analytic problem. No result of this appendix opens its downstream admission exit.

Declarations

Mathematical contribution. The release is the explicit prescribed-background finite model, its calibration correction, its complete destructive instrument, its locality certificate, and its target-adapted null transport protocol. The appendix proves a classical lapse-composition formula and a compact rod-detection witness under stated hypotheses.

Physical scope. The construction supplies a continuum classical metric and finite operational semantics. It does not supply microscopic within-tick dynamics, a quantum field-theoretic scaling limit, gravity-generated entanglement, a Page curve, a horizon, an island, or holographic emergence.

Source scope. Cheung's manuscripts are cited with identifiers and supplied versions. Source references motivate comparisons; they are not automatic transfers of models or proof hypotheses. Bibliographic descriptions for internal manuscripts are not invented publication titles or dates. The introductory philosophy and the directly accessed companion records are documentary sources, not a proof-checking service.

Analytic scope. No full inherited characteristic composition, joint section, transverse estimate package, contact gluing, or quantum output wavefront theorem is constructed. No inherited quantum coefficient is admitted.

XV-B's downstream exit is not opened, XVI-N is not discharged, and O.1 remains NOT_CLOSED within this release.

Verification scope. The source is self-contained for pdfLaTeX and requires no external figures or bibliography database. No successful compilation, proof-assistant check, independently executed symbolic calculation, laboratory realization, referee approval, or journal acceptance is asserted.

References

N. Cheung, *The Philosophy of Enter: Destructive Readout, Quantum Recovery, and the Refusal of Identification*, EntPhil-v2.tex, version 2. Introductory essay, Enter Calculus Companion. https://enter.grok.me/enter. Documentary orientation, not a theorem premise.

N. Cheung, *Manuscript I*, I.tex, version 2. Supplied manuscript excerpts, Enter Calculus Companion. Relevant items: destructive Enter characterization, pure finite configurations, name ownership, and the distinction between record and residual compatibility. https://enter.grok.me/.

N. Cheung, *Observation Boundaries for Destructive Quantum Readout and External Geometry*, En03-A.tex, version 1. Enter Calculus Companion. Relevant items: four geometric interface structures, three readout interfaces, and prescribed-network locality. https://enter.grok.me/browse/En03-A.

N. Cheung, *Manuscripts VII-E and VII-F*, VII-E.tex and VII-F.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant items: fixed conical geometry, oriented holonomy, observation boundaries, and VII-F Theorem thm:event. https://enter.grok.me/.

N. Cheung, *Manuscript VIII-B*, VIII-B-v3.tex, version 3. Supplied manuscript excerpts, Enter Calculus Companion. Relevant item: Theorem thm:dictionary, the variable-deficit spinless cone dictionary. https://enter.grok.me/.

N. Cheung, *Manuscript XIX-A*, XIX-A.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant items: separation of time prescriptions and Theorem thm:rod-detection. https://enter.grok.me/.

N. Cheung, *Realization of Relational Time by Destructive Enter on a Clock Register*, En09-A.tex, version 1. Enter Calculus Companion. Relevant items: complete clock instrument, Propositions prop:no-return, prop:record-not-residual, and prop:no-ccr. https://enter.grok.me/browse/En09-A.

N. Cheung, *Manuscript VIII-A*, VIII-A-v3.tex, version 3. Supplied manuscript excerpts, Enter Calculus Companion. Relevant item: complete instrument agreement versus classical record agreement. https://enter.grok.me/.

D. N. Page and W. K. Wootters, ``Evolution without evolution: Dynamics described by stationary observables,'' *Physical Review D* **27** (1983), 2885–2892. https://doi.org/10.1103/PhysRevD.27.2885. Background comparison; the finite extraction identities in this paper are proved explicitly.

N. Cheung, *Manuscript En07-A*, En07-A.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant item: Proposition prop:no-landauer-from-channel. https://enter.grok.me/.

N. Cheung, *Manuscript En05-A*, En05-A.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant item: Theorem thm:cone, complete schedule locality and metric envelopes. https://enter.grok.me/.

N. Cheung, *Manuscript En06-A*, En06-A.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant item: Theorem thm:order, complete wired-network ancestry and its influence bound. https://enter.grok.me/.

N. Cheung, *Manuscript En08-A*, En08-A.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant items: Theorem thm:main, the scheduler obstruction, and the conditional cone-representation statements. https://enter.grok.me/.

M. Ito and B. F. Lourenco, *The p-cones in dimension $n \geq 3$ are not homogeneous when $p \neq 2$*, arXiv:1701.05801. https://arxiv.org/abs/1701.05801. Source for the homogeneous strictly convex cone classification; not a construction of an operational selector.

N. Cheung, *Manuscript XXII-F*, XXII-F.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion.

Relevant items: conditional assembly and completion obligations F1–F6. https://enter.grok.me/.

N. Cheung, *Resource-controlled operational approximation of null boundary transport*, En08-B.tex, version 1. Enter Calculus Companion. Relevant items: prescribed metric and speed, lattice approximation, reference-preserving SWAP identity, resources, and error accounting. https://enter.grok.me/browse/En08-B.

N. Cheung, *Manuscript VIII-C*, VIII-C-v3.tex, version 3. Supplied manuscript excerpts, Enter Calculus Companion. Relevant item: Proposition prop:not-einstein. https://enter.grok.me/.

N. Cheung, *Manuscripts Oai282-A and Oai282-B*, Oai282-A.tex and Oai282-B.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Local Ward comparisons, not Einstein dynamics or verification of an OpenAI catalogue family. https://enter.grok.me/.

N. Cheung, *Manuscript XXII-E*, XXII-E.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant item: Theorem thm:flat; used here only for spacetime-constant affine representatives. https://enter.grok.me/.

N. Cheung, *Manuscript Oai365-A*, Oai365-A.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant items: Propositions prop:signature and prop:lapse. The comparison manuscript is distinct from the separately attributed OpenAI catalogue. https://enter.grok.me/.

N. Cheung, *Manuscript III*, III.tex, version 2. Supplied manuscript excerpts, Enter Calculus Companion. Classical equation-space and descent distinctions. https://enter.grok.me/.

N. Cheung, *Manuscript VI*, VI.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant item: Theorem thm:descent, classical finite-time harmonic relational descent. https://enter.grok.me/.

N. Cheung, *Original-Rod Lapse: Classical Freeze and Quantum Admission Contract*, XX-A.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant items: Theorem thm:lapse, original-rod classical second-order coefficients, and the separate quantum admission contract. https://enter.grok.me/.

N. Cheung, *Manuscript Em02-E*, Em02-E-v1.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant items: Theorem thm:lapse, classical relational lapse through second order, and the incomplete quantum regulator-removal audit. https://enter.grok.me/.

N. Cheung, *Manuscripts XIV-I and XIV-J*, XIV-I.tex and XIV-J.tex, version 1. Supplied manuscript excerpts, Enter Calculus Companion. Relevant items: XIV-I's constructed internal amplitude at I2; the missing full characteristic map at I3/J2; XIV-J's reconstruction-descent criterion, supported-sector off-characteristic reference, and no-admission conclusion. https://enter.grok.me/.

N. Cheung, *Supplied technical and standing-boundary records: XV-B, XVI-N, and the O.1 ledger*, including XV-B.tex, version 2, and XVI-N.tex, revision r12. Enter Calculus Companion. Documentary scope only: existence of technical interfaces is not downstream quantum admission, and no gate is discharged by the finite model of this paper. https://enter.grok.me/.